

University of Alberta

THREE-DIMENSIONAL FORCE SYSTEMS FROM ORTHODONTIC
RETRACTION APPLIANCES

by

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A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**.

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Abstract

Numerical simulations have previously been used to model the force systems produced by orthodontic retraction appliances; however, these results have not been extensively validated in a 3D setting. The purpose of this study is to evaluate the accuracy of numerical predictions in a 3D setting. Empirical and numerical techniques are also used to compare several orthodontic retraction springs using characteristics such as force, moment, and moment-to-force ratio.

The errors in the numerical predictions of the force systems for 3D activations are found to be more pronounced than for planar activations. The most significant sources of error are found to be helices in the spring and placement of the spring in the mounting brackets.

Comparison amongst spring designs shows that the archial-loop design is clinically more favorable than the vertical-loop and comparable to the T-loop. The initial curvature of the end sections in the archial-loop design significantly increases moment-to-force ratio.

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Chapter 1

Introduction

Malocclusion is defined as the misalignment of teeth and/or incorrect relation between the teeth of the maxillary (upper) and mandibular (lower) dental arches. A common cause of this malocclusion is the crowding of teeth. To deal with this crowding, one approach employed by orthodontists is to extract a posterior tooth to create more space in the dental arch ($\sim 6 - 9$ mm results from the extraction of a bicuspid). Subsequently, this extra space provides an area to be used for tooth relocation during orthodontic treatment. It is this tooth displacement which then results in improved occlusion. For this reason, space closure between teeth is an important orthodontic task.

Without effective space closure tooth extraction may not improve occlusion or benefit the patient. This is because excess space in the dental arch can make it more difficult to masticate food and maintain oral hygiene. In addition to this, the extra space created from tooth extraction is often undesirable for purely aesthetic reasons. This is further justification for the need for effective space closure following tooth extraction.

Like all orthodontic treatment, space closure involves the clinical application of several biomechanical concepts. Concepts such as: the understanding of the methods of application of specific force systems to a single tooth or a selected group of teeth (from here on referred to as the tooth), the knowledge of the mechanical principles governing the generation of forces and moments, and the comprehension of how forces and moments will affect the tooth, anchorage, and surrounding periodontium are all key to successful orthodontic treatment. In fact, it is the goal of the clinician to minimize or eliminate unknown elements of the orthodontic treatment through the application of these concepts. Through this application, the specific force systems needed to bring about the desired type, direction, and rate of tooth movement can be achieved by inducing the appropriate stresses and strains on the tooth and its supporting

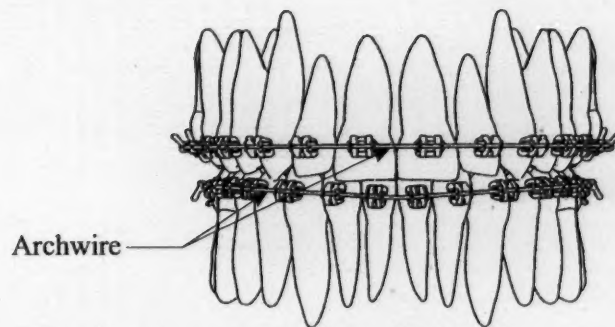


Figure 1.1: Sliding Mechanics Schematic

structure.

To induce the stresses and strains necessary for effective space closure, two general techniques are currently employed by orthodontists. These techniques are generally known as sliding mechanics and segmented mechanics. In sliding mechanics, tooth movement is controlled by the presence of a relatively rigid archwire which maintains the proper translational and rotational orientation / position of the tooth as shown in Figure 1.1. Orthodontic brackets are bonded to either the buccal or lingual sides of selected teeth. These brackets are then ligated to an archwire which is fixed along the orthodontic arch. A coil spring or elastic is subsequently used to cause the tooth to slide along the archwire. A problem that arises with sliding mechanics, however, is the presence of friction between the archwire and bracket. This friction causes sliding mechanics to be a statically indeterminant problem. Therefore, the actual amount of force applied to the tooth cannot be determined. Furthermore, the friction that is present is dependant on not only how the forces are applied but also on the materials used and the surface finishes of these materials. Both experimental (Drescher, D et al., 1989; Braun, S et al., 1999) and numerical (Kojima, Y and Fukui, H, 2005) studies have been conducted to investigate this friction; however, there is still no systematic method to determine these unknown friction forces.

In segmented mechanics a "spring", typically a section of wire initially deformed to a specific shape (Figure 1.2(a)), is used to apply specific moments and forces (Figure 1.2(b)) to the brackets attached to several teeth of the dental arch (hence "segments" of teeth). The specific force systems at each spring attachment control the movement of the tooth or segments. This is one of

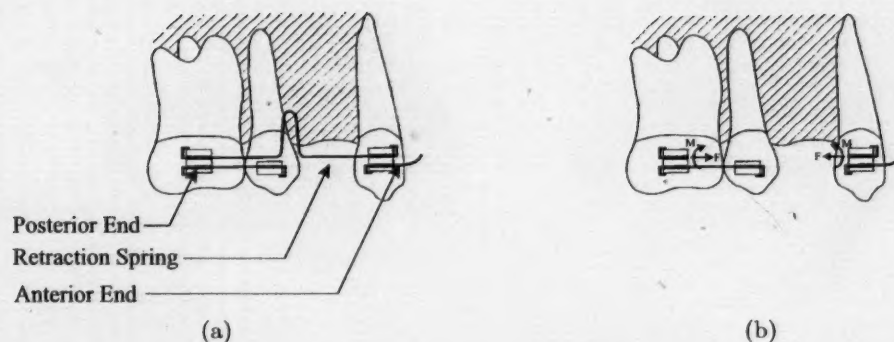


Figure 1.2: Segmented Mechanics Schematics (Reprinted from Raboud, D et al. (1997) with permission from American Association of Orthodontists)

the advantages (Braun, S and Marcotte, MR, 1995) of segmented mechanics because the force systems applied to the teeth are determinate systems. A disadvantage, however, is that undesired tooth movements can occur due to the lack of a rigid archwire. Still, with a proper understanding of the mechanics involved, this movement can be controlled. For this reason, both experimental and numerical studies have been conducted as a means of determining the proper forces and moments needed to control this movement. Investigation into the types of appliances that can produce these force systems has also been conducted.

To fully appreciate the factors affecting this tooth movement, one must consider both the tooth and its supporting structure (the periodontal ligament, blood vessels, extracellular fluid, etc.). This is because the supporting structures resist the movement of teeth and generate an effective resistance that determines the force system needed to properly relocate the tooth. This effective resistance force acts through a point known as the center of resistance (C_{res}). Shown in Figure 1.3, C_{res} is the point at which if a single force was applied, the tooth would translate in the direction of the force. If a force is not applied at the center of resistance, as is the case when the force is applied at the bracket on the tooth, rotation of the tooth occurs about a point known as the center of rotation (C_{rot}). This rotation occurs because the applied force (at the bracket) and the effective resistance force (at C_{res}) form a couple on the tooth. The location of C_{rot} (see Figure 1.4) is dependant on the force system applied. To counter any undesired rotation or couple, an opposing moment must be applied to the bracket by the orthodontic appliance.

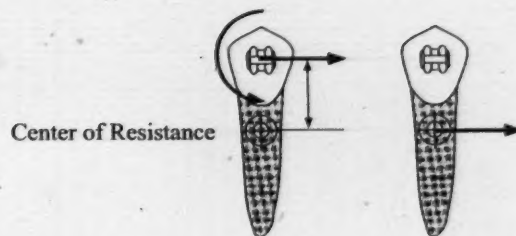


Figure 1.3: Location of Center of Resistance (Reprinted from Nanda, R (1997) with permission from Elsevier Inc.)

Several specific types of tooth movements are shown in Figure 1.4. Translation of a tooth is defined as movement that involves the translation of the root apex and crown of the tooth equal distances in the same horizontal direction. This is a result of a uniform stress at the interface of the root and periodontal ligament (Nanda, R, 1997) as shown in Figure 1.5(a). To generate this stress distribution, the appropriate moment-to-force (M/F) ratio (the ratio of the applied moment to the applied force acting at the bracket) must be applied to the tooth. However, because the size, shape, and length of the roots of teeth are different for each tooth, the M/F ratio that leads to translation varies for the different teeth. Furthermore, translation of segments of teeth may also be desired. This affects the required M/F ratio as well. While there is some discrepancy in the M/F ratio values that produce translation, Table 1.1 shows some examples of M/F ratios that are believed to produce translation. For the purpose of this work, the value of 8.5 mm will be used assumed to produce translation.

Translation of a tooth is shown in Figure 1.4(c); however, tipping of the tooth is sometimes desired. Controlled tipping occurs when the crown of the tooth moves a larger distance than the root. Controlled tipping (shown in Figure 1.4(b)) occurs under the stress distribution of the form shown in Figure 1.5(b). Root movement occurs when the root moves a larger distance than the crown. This is shown in Figure 1.4(d) and is a result of the stress distribution shown in Figure 1.5(d). In both of these cases C_{rot} is located either apical or gingival to C_{res} . Uncontrolled tipping is the limiting case and it results when a stress distribution such as that shown in Figure 1.5(c) occurs. As shown in Figures 1.4(a) and 1.4(e), C_{rot} is located at or near C_{res} .

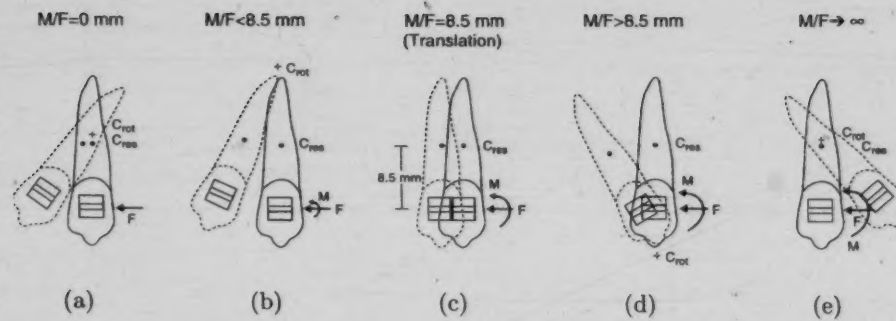


Figure 1.4: Movements for Various M/F Ratios (Reprinted from Raboud, D et al. (1997) with permission from American Association of Orthodontists)

Table 1.1: Translational Moment-to-Force Ratios for Various Teeth

Tooth	Maxillary						
	1	2	3	4	5	6	7
M/F	8.2	7.7	9.4	8.6	8.6	8.4	7.6
Tooth	Mandibular						
	1	2	3	4	5	6	7
M/F	7.6	8.4	10.2	8.6	8.6	8.4	7.1
Segment	Maxillary						
	21 / 12	321 / 123	653 / 356	6543 / 3456	76543 / 34567	65 / 56	765 / 567
M/F	8.0	8.4	8.8	8.8	8.6	8.5	8.2
Segment	Mandibular						
	21 / 12	321 / 123	653 / 356	6543 / 3456	76543 / 34567	65 / 56	765 / 567
M/F	8.0	8.8	9.1	9.0	8.6	8.5	8.2

Values taken from Siatkowski, RE (1994)

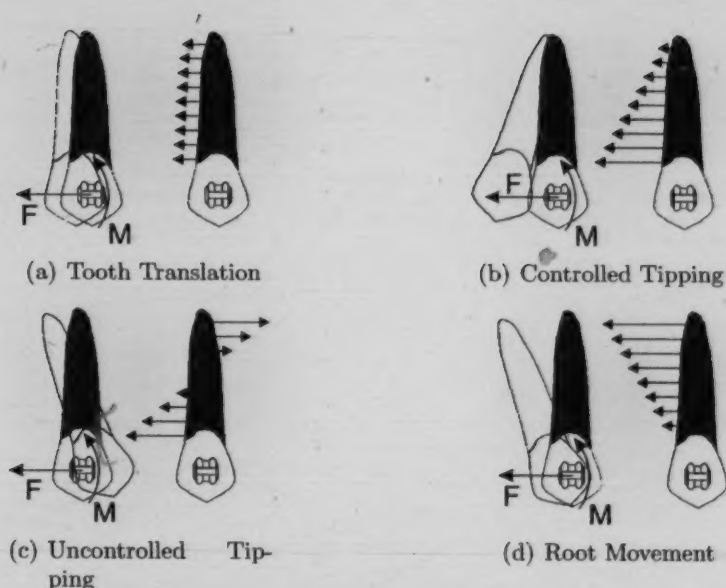
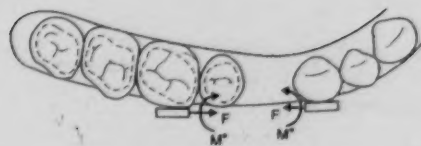


Figure 1.5: Tooth Movement Stress Distributions (Modified from Nanda, R (1997) with permission from Elsevier Inc.)

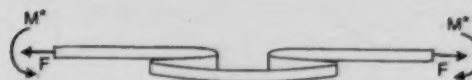
This concept of M/F ratio does not only apply with regards to tooth tipping. M/F ratio can also be applied to tooth rotation. That is, since the force of the spring is applied at the bracket, a rotation of the tooth about its long axis can be created. This is illustrated in Figure 1.6 for a retraction spring mounted on the outer or buccal side of the teeth. Figure 1.6(d) shows that, if a M/F ratio (denoted as M^*/F in the occlusal plane) of 3.5 mm is applied to the tooth, translation of the tooth occurs. Figures 1.6(c) and 1.6(e) show the corresponding tooth rotation for M^*/F ratios below and above 3.5 mm respectively.

1.1 Literature Review

Several studies have been conducted to help better understand the biomechanical concepts involved in segmented mechanics. These studies, both numerical and experimental, have been employed to help understand results, model scenarios, and/or improve patient treatment. This section gives a brief overview of various studies that have been conducted.



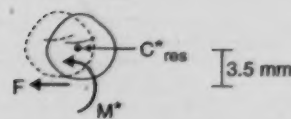
(a) Occlusal View



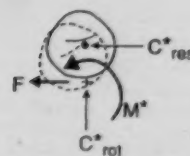
(b) Top View of Spring



(c) $M^*/F=0$



(d) $M^*/F=3.5\text{mm}$ (Translation)



(e) $M^*/F < 3.5\text{mm}$

Figure 1.6: M/F Ratio: Occlusal View (Reprinted from Raboud, D et al. (1997) with permission from American Association of Orthodontists)

1.1.1 Empirical Studies

Many studies have tried to experimentally determine the forces and moments generated by orthodontic retraction appliances. While developed at different times and by different people, all of the techniques are inherently similar. To measure the forces and moments produced by the spring, a force-torque transducer is used. More specifically, a calibrated set of strain gauges is used to measure strain which is then converted to forces and moments. To measure the displacement of the retraction spring, linear variable displacement transducers (LVDTs), vernier gauges, or any other form of displacement measurement are used. Examples of studies conducted using such techniques are: Burstone, CJ et al. (1976); Solonche, DJ et al. (1977); Chen, J et al. (2000); Ferreira, M et al. (2005). However, these studies were limited to planar force systems. That is, only the horizontal and vertical forces in the plane of the spring as well as a torque about an axis perpendicular to the plane of the spring were analyzed. While these planar force systems are the most clinically applicable, and are often sufficient, natural progression has led to the measurement of full three-dimensional force systems to better understand and model the biomechanics of retraction appliances. Faulkner, MG et al. (1989); Bourauel, C et al. (1992); Planert, J (1992) all developed apparatuses for this purpose. These apparatuses were capable of measuring forces and torques in all three spatial directions and thus, allow a more complete biomechanical analysis of the force systems.

This ability to measure both the planar and three-dimensional force systems has also led to empirical studies involving retraction spring geometry, material selection, and spring placement. Geometrical studies have involved using the aforementioned apparatuses to aid in the design of new orthodontic retraction springs (Ferreira, M et al., 2005) and to further investigate existing retraction spring designs (Menghi, C et al., 1999; Chen, J et al., 2000; Katona, TR et al.). This research has also furthered studies investigating the desired or optimum force needed for orthodontic treatment (Burstone, CJ and Koenig, HA, 1976; Quinn, R and Yoshikawa, D, 1985; Tanne, K et al., 1988).

1.1.2 Numerical Studies

To numerically model the forces from orthodontic retraction appliances, various techniques have been employed. Finite element analyses (Yang, TY and Baldwin, JJ, 1974; Faulkner, MG et al., 1989; Chen, J, 1994), finite difference formulations (Koenig, HA and Burstone, CJ, 1974), and segmental techniques (Lipsett, A et al., 1990) have all been employed to model the force systems.

Stil, similar to the empirical studies, these numerical studies were limited to planar force systems.

While not as prevalent, three-dimensional (3D) numerical studies have also been conducted. Raboud, D et al. (1997) and Raboud, D et al. (2001) both implement a three-dimensional segmental approach (Raboud, D et al., 1996) that models three-dimensional force systems. For example, out-of-plane pre-activated bends were investigated by Raboud, D et al. (1997).

For the planar cases, extensive verification of both numerical and experimental results has been conducted. Faulkner, MG et al. (1989); Lipsett, A et al. (1990); Koenig, H et al. (1980) all verified their numerical procedures by incorporating experimental testing with the numerical modeling. This showed that beam theory can be accurately applied to planar scenarios. However, in the three-dimensional situations no work has been documented that verifies the results of this three-dimensional numerical method to that of clinical or experimental results. This verification is needed to validate the assumptions used in deriving this numerical method. Assumptions such as the presence of thin curved beams and rigid boundary conditions are two such examples. This is because it is known that these assumptions are not satisfied in the empirical cases.

1.2 Thesis Outline

The primary purpose of this study is to investigate the accuracy of the numerical prediction of these orthodontic force systems generated in a full three-dimensional setting. Also investigated, however, are the force systems generated from a relatively new retraction spring design (an archial-loop spring) and how this design compares to other common designs.

The materials and methods used to measure and model the 3D force systems from the retraction appliances are outlined in Chapter 2. Both the experimental apparatus and the experimental testing procedure are described. A summary of the numerical modeling procedure is also included.

In Chapter 3, details of the results of both the experimental tests and numerical simulations are presented. Differences between the results of inter- and intra- spring testing are discussed. Comparisons between the empirical and numerical results are also used to evaluate the validity of the numerical procedure. Both planar and non-planar activations are used as well as initially planar and non-planar springs.

An in-depth study of the archial-loop orthodontic spring is discussed in Chapter 4. The study compares the archial-loop spring to the more common

T-loop and vertical-loop springs designs. Clinically relevant scenarios are evaluated and used to discuss the advantages and disadvantages of the archial-loop spring relative to the vertical-loop and T-loop spring.

The conclusions of this thesis are presented in Chapter 5. In this chapter the advantages and limitations of the testing technique are discussed. Suggestions for further work are also presented.

Chapter 2

Materials and Methods

This chapter outlines the experimental and numerical procedures used to determine and simulate the three-dimensional force systems generated from orthodontic retraction appliances. In addition, basic orthodontic and engineering terminology and concepts are defined, descriptions of the experimental equipment are provided, explanations of the experimental testing techniques are given, and a summary of the numerical modeling procedure is presented.

2.1 Basic Terminology and Concepts

Standardized terminology exists in orthodontics and engineering that allows for the consistent identification and description of various locations, structures, conditions, and movements. For example, terms such as buccal and tipping describe a specific direction and type of movement in the human mouth respectively. This terminology section is therefore intended to clarify these terms for the use in this thesis.

Like any three-dimensional space, the human mouth can be described by an ortho-normal coordinate system. Figure 2.1 illustrates the meanings of the common orthodontic terms buccal / lingual, mesial / distal, and anterior / posterior. The terms apical and gingival refer to the directions in and out of the page on Figure 2.1 with apical towards the crown of the tooth and gingival towards the root of the tooth. A three-dimensional reference framework can also be set relative to the teeth. This is shown in Figure 2.2 with the orientations of the sagittal, occlusal, and frontal planes relative to the head, tooth, and bracket.

As outlined in Chapter 1, several classifications of tooth movement exist - translation, controlled tipping, root movement, uncontrolled tipping. These

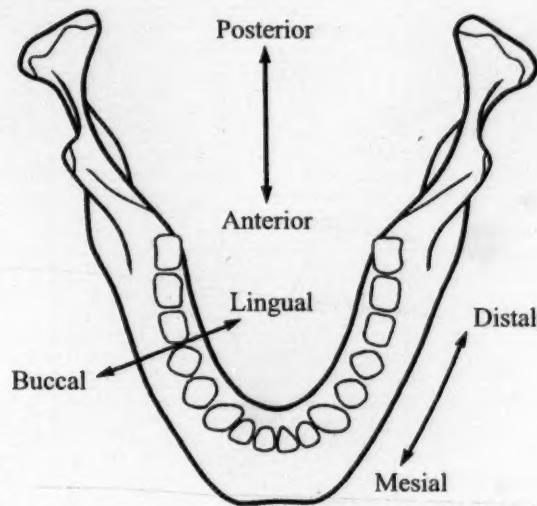


Figure 2.1: Orthodontic Directions

movements are controlled by the moment-to-force (M/F) ratio generated by the orthodontic appliance. It should be noted, however, that while the goal of orthodontic treatment is to create this desired tooth movement, the concept of static equilibrium still applies since the motion of the teeth is essentially quasi-static. Therefore, the inertial properties of the tooth do not affect results.

As it is the basis for the numerical model used in this thesis, the term thin beam is also relevant to this work. While a beam can be defined as any slender body that has cross-sectional dimensions much less than its length, a thin curved beam is further constrained. A thin curved beam is constrained such that the radial cross-section dimension (the dimension parallel to the radius of the bending) is much less than all radius of curvatures in the initial and deformed geometries. This ensures that the bending is primarily due to bending moments and not shear forces.

One final term that will be used throughout this thesis is the term boundary conditions. Boundary conditions refer to how the beam is fixed or attached at its ends. For example, can the beam pivot or slide at its ends or is it rigidly attached. In the case of orthodontics, this would refer to how the spring is attached at the mounting brackets.

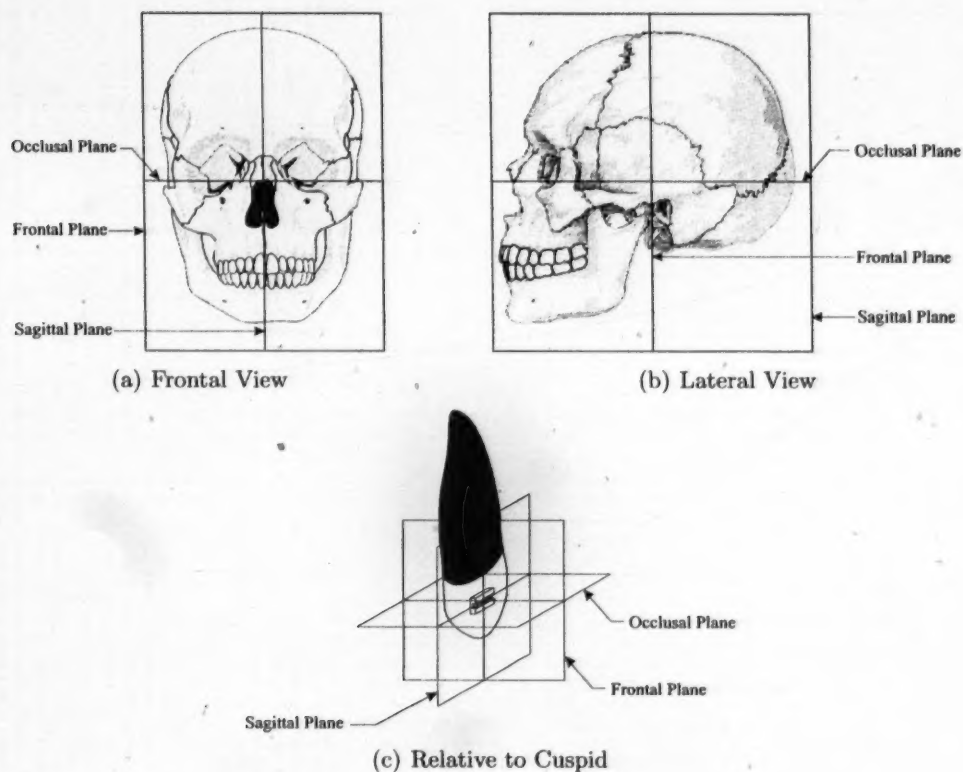


Figure 2.2: Orthodontic Planes in Human Skull

2.2 Orthodontic Springs

2.2.1 Spring Designs

Various sizes and shapes of segmented mechanics orthodontic springs are currently being used by practicing orthodontists. This study evaluated two common designs. These are the T-loop spring (T-spring) and vertical-loop spring (V-spring) shown in Figure 2.3. A third spring design, which is a modification of the vertical-loop spring, was also investigated. This design is shown in Figure 2.3(c) and is known as an archial-loop spring (A-spring). It is not as commonly used by practicing orthodontists.

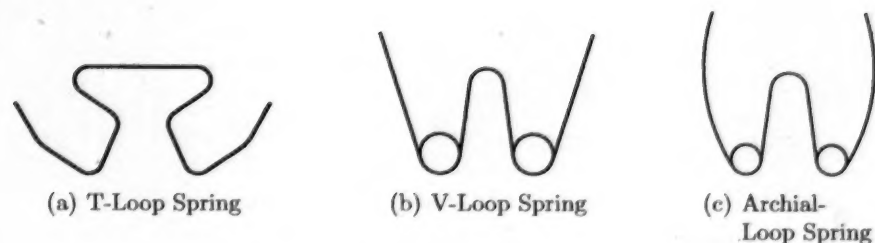


Figure 2.3: Orthodontic Spring Designs

2.2.2 Geometry Measurement

The geometry of the various spring designs must be known in order to numerically model the forces and moments generated during activation (stretch) of the springs. Radii, lengths, angles, and wire cross-section size and shape are all needed. To determine this information, and create a digital model of each of the springs, each spring was scanned with a resolution of 1200 dpi. Computer software was then used to measure individual lengths and radii of each section of spring from these scans. However, because only linear measurements could be made, the radii of the curved sections were determined using $R = \frac{1}{8h}(c^2 + 4h^2)$. Figure 2.4 illustrates what each symbol represents in this equation while Appendix B contains the derivation. Manufacturer specifications were used for the cross-section size and shape but were confirmed by manual measurement with a caliper of accuracy ± 0.01 mm.

2.2.3 Material Properties

Orthodontic springs can be constructed from a wide arrangement of materials. For example, materials such as stainless steel, nickel-titanium alloys (NiTi), and titanium-molybdenum alloys (TMA) are commonly used to make both orthodontic wires and springs. Understanding material properties is therefore essential when selecting, using, and modeling such wires. For the purpose of this study, the intrinsic material properties that were required to model the orthodontic springs were: modulus of elasticity (Young's modulus), shear modulus (modulus of rigidity), and yield stress (elastic limit). Such properties

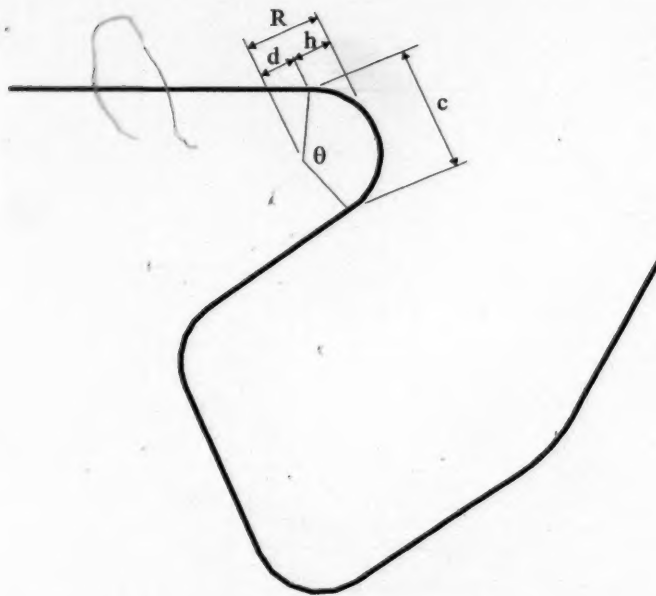


Figure 2.4: Curved Section Radius Calculation Variables

can be determined from material data specifications and verified with tensile and torsion tests.

2.3 Experimental Apparatus

Experimental testing of the orthodontic spring three-dimensional force systems was performed using the experimental apparatus shown in Figure 2.5. The apparatus consists of three Mitutoyo Digimatic Indicators with a resolution of 0.01 mm and a range of 30 mm. These indicators were used to measure the x , y , and z locations of the center pedestal on which one of the spring's mounting brackets was anchored. The other mounting bracket, to which the opposite end of the spring was mounted, was stationary and was bolted to a Nano17 multi-axis force/torque sensor system (unit FT60086, SI-50-0.5). This transducer, made and supplied by ATI Industrial Automation, was distributed complete with the user-interface and calibration software capable of simultaneously measuring forces in the x , y , and z directions (see Figure 2.6)

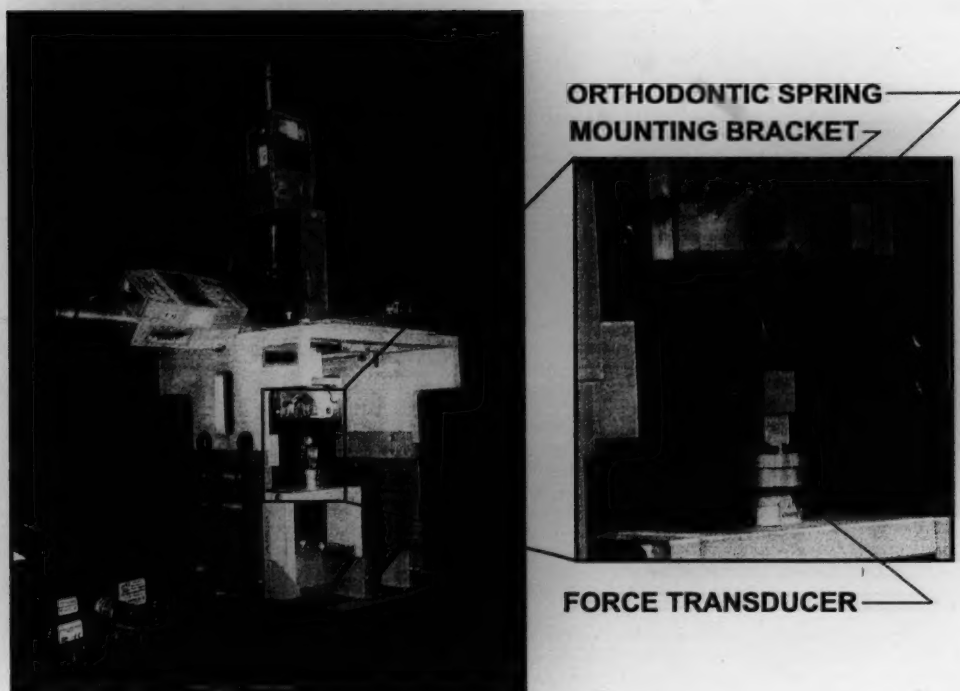


Figure 2.5: Experimental Apparatus

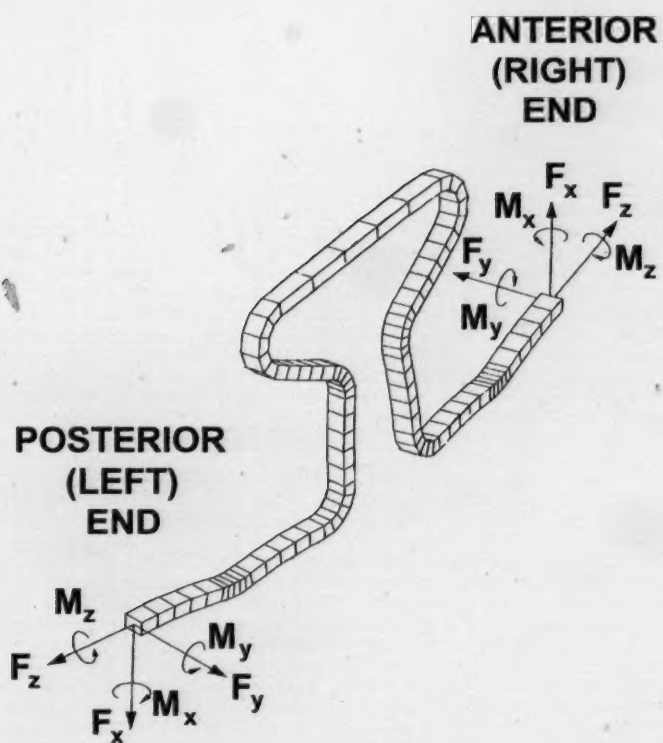


Figure 2.6: Three-Dimensional Orthodontic Force System

and moments about the x , y , and z axes. The transducer was capable of measuring forces with a resolution of 0.001 N and moments with a resolution of 0.01 Nmm. An ATI Industrial Automation DAQ/FT Interface/Power supply was also used to power the transducer. The Interface/power supply was directly linked to a National Instruments Data Acquisition System (NI-DAQ) in a personal computer so that empirical data could be directly recorded.

2.4 Calibration

Prior to testing the springs, calibration of the Nana17 force transducer was required. Calibration was achieved using the customized self-calibration software distributed with the sensor. To check this calibration, known masses were used as test weights to verify load cell accuracy. Known masses of 25 g, 50 g, and 100 g were used to test the sensor in both compression and tension. To test moment accuracy, each mass was hung from the sensor at a known offset from the sensor's z -axis. Knowing the mass and moment arm, moment accuracy could then be verified.

2.5 Experimental Testing

To test the springs in the experimental apparatus, each spring was anchored into the brackets by bending the excess wire around the bracket as shown in Figure 2.7. This prevented the spring from sliding in the bracket when the spring was in tension and is comparable to procedures used in orthodontic practice. No testing was performed for any springs in compression. Initially, each spring was brought into its neutral position prior to activation. Neutral position is defined as the distance between the two mounting brackets when the ends of the spring arms are colinear and have no axial force applied. Figure 2.8 illustrates the neutral position of a vertical-loop spring. From the neutral position, the springs were activated (stretched) to various activations (0 – 5 mm) or inter-bracket distances (IBDs). Inter-bracket distance is defined as the linear distance between the 2 mounting brackets. All activations were kept below the elastic limit (determined by the numerical procedure) of the spring.

Activations were initially kept in-plane to verify results of the testing method. That is, with the ends of the spring colinear, each spring was activated along the z -axis (see Figure 2.6). This test was repeated 3 times for each of the 6 springs (2 T-springs, 2 V-springs, and 2 A-springs). Results from the planar tests were then compared to the numerical simulations. Numeri-

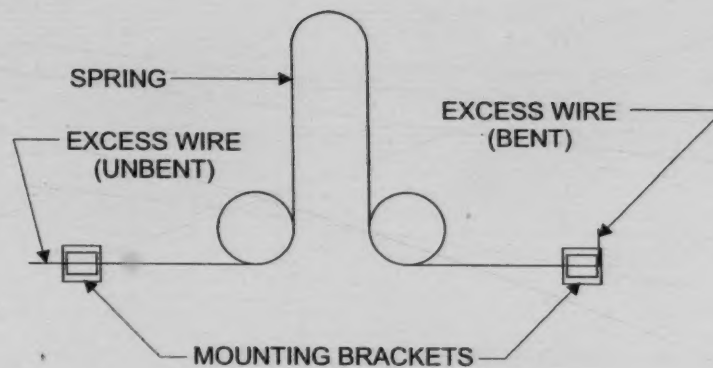


Figure 2.7: Orthodontic Spring in Mounting Brackets

cal simulations were conducted using the digitized geometries for each of the springs at the same activations used experimentally.

Since the testing apparatus only included one force transducer, two sets of tests were required for each spring. One test was required with the right (mesial / anterior) end of the spring mounted to the force transducer and one was required with the left (distal / posterior) end of the spring mounted to the force transducer. Numerically, the moments and forces could be calculated at each end of the spring simultaneously.

Non-planar activations were used to investigate three-dimensional force systems. These non-planar activations were accomplished by displacing one end of the spring relative to the other in combinations of x , y , and z displacements. Displacements of 1, 2, and 3 mm were used in the x and y directions while z activations were kept below the elastic limit. Similar to the planar tests, 3 tests were conducted for each scenario.

2.5.1 Spring Placement

As previously mentioned, to place the springs in the mounting brackets of the experimental apparatus the end sections were inserted through the slots and anchored by bending the excess material around the bracket. Once placed in the mounting brackets, the lengths of the end sections of the spring (spring arms) were measured. This measurement was conducted prior to spring deformation. The edge of the mounting bracket was taken to be the end of the spring arm as shown in Figure 2.8. For this measurement, a caliper and ruler were used because no convenient digital measurement was possible. The

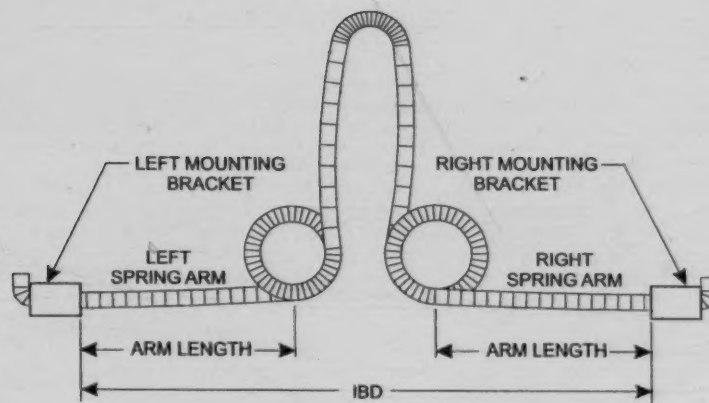


Figure 2.8: Neutral Position

lengths, radii, and angles of all other sections determined from the digital scan were still used.

2.6 Numerical Modeling

Numerical simulations of the three-dimensional force systems generated by the activation of these orthodontic retraction springs were performed using a previously described numerical method (Raboud, D et al., 1996). This nonlinear procedure uses thin beam theory, divides the springs into several segments, and applies force and displacement compatibility between the segments to solve the resulting system sequentially along the rod. Previous studies (Raboud, D et al., 1997, 2001) have implemented this technique to investigate both two- and three-dimensional force systems generated from orthodontic retraction springs; however, to date the three-dimensional results have not been verified experimentally.

2.7 Spring Repeatability Study

Five T-loop springs and 5 archial-loop springs were investigated – as a separate study to the three-dimensional force system modeling – to determine the orthodontist's ability to bend identical springs. This involved determining the geometry of each spring and conducting force / moment measurements with each spring to determine the differences in behavior. As it is the planar situa-

tions which are of more interest in clinical orthodontics, the repeatability tests were restricted to planar activations. Furthermore, instead of finding the neutral position of the spring and then activating the spring outwards along the z-axis, measurements were recorded for known inter-bracket distances (IBDs). This simulated orthodontic treatment. Three tests were conducted for each scenario. To increase the number of scenarios that each spring could be tested in, several different spring arm lengths (see Figure 2.8 for an illustration of the spring arms) were used. That is, once the tests for a given arm length were completed, the spring arms were drawn further through the mounting bracket (making the arms shorter) and additional tests were completed. Successive tests were, however, restricted to shorter arm lengths because the excess wire that was bent around the mounting bracket was plastically deformed and could no longer be used as part of the spring.

Chapter 3

Results and Discussion

In this chapter empirical and numerical results are shown for both two- and three-dimensional activations of several orthodontic appliances. Comparison of the experimental testing technique and numerical modeling procedure is presented. Also included in these results is the spring geometry and material properties determined for each spring. These results are important as they directly affect the accuracy of the numerical modeling. The benefits and limitations of the spring designs are also discussed.

3.1 Material Properties

Each spring was constructed from TMA with a nominal cross-section of 0.432×0.635 mm ($0.017'' \times 0.025''$). These cross-section dimensions were confirmed to within ± 0.01 mm at various location on the spring using a digital caliper. Using these cross-section dimensions with the results from 3 separate tensile tests (shown in Figure 3.1), Young's modulus for the TMA was determined to be 62.5 ± 0.7 GPa. This is 9.4% lower than the specified value of 69 GPa. Other material properties that were used for the TMA are shear modulus (26.5 GPa) and yield stress (1.24 GPa). All values were obtained from material specifications.

While not a material property, the torsion factor of the spring wire was also required. The torsion factor is a dimensionless property, based on the height and width of the rectangular cross-section, that is used in determining the rotational stiffness of the wire under torsion. For a cross-section of 0.432×0.635 mm, the torsion factor is 0.193¹.

¹Value obtained from linear interpolation of values presented in torsional parameter table. See Boresi, A et al. (1993) (pp. 252)

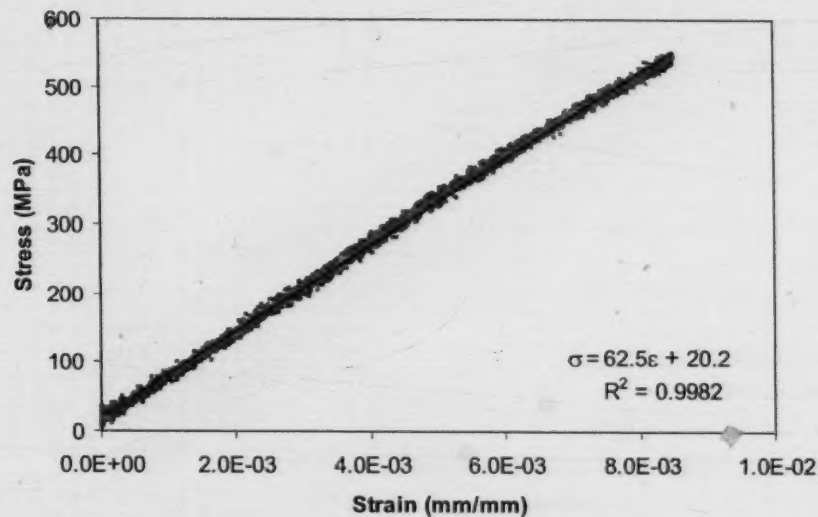
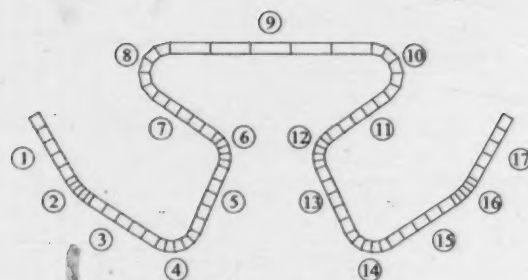


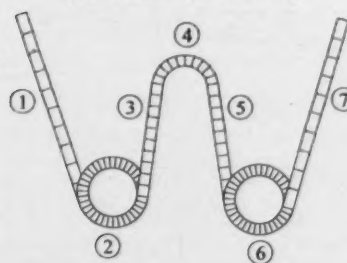
Figure 3.1: Results - TMA Tensile Test

3.2 Spring Geometry

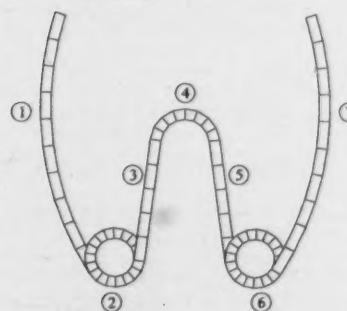
Figures 3.2(a), 3.2(b), and 3.2(c) show the digitized geometries of a typical T-loop spring, vertical-loop spring, and archial-loop spring respectively. The section numbers in these figures correspond to sections that were measured experimentally and used to digitize the spring geometry for numerical modeling. Both curved and straight sections were used in the digitization process. To define a curved section a radius and an angle were required; to define a straight section only a length was required. The measured geometries of the springs are shown in Tables 3.1-3.5. Tables 3.1 and 3.2 show the geometries of 5 T-springs and 5 A-springs respectively. These are the springs that were used in the repeatability study. "Variable" is indicated for the length and angle for sections 1 and 17 or 1 and 7 to represent that these sections are the springs arms which are dependent upon the spring placement in the mounting brackets. These springs of similar design were bent by an orthodontist and were intended to be clinically identical springs; however, Tables 3.1 and 3.2 show that there is some variability between the springs. Tables 3.3, 3.4, and 3.5 show the geometries of the planar T-springs, V-springs, and A-springs respectively that were used in the planar and non-planar analyses.



(a) T-Spring Geometry



(b) Vertical-Loop Spring Geometry



(c) Archial-Loop Spring Geometry

Figure 3.2: Orthodontic Springs Geometries

Table 3.1: Repeatability Test: T-Loop Spring Geometries

Spring	Section 1 (mm)	2 (mm) (°)	3 (mm)	4 (mm) (°)	5 (mm)	6 (mm) (°)	7 (mm)	8 (mm) (°)	9 (mm)
T3	variable	2.42 32.6	3.89	1.05 104.0	2.79	0.85 74.6	3.20	1.00 139.3	7.88
T4	variable	1.14 25.6	4.01	1.09 111.2	3.19	1.02 74.4	3.31	0.91 139.7	7.94
T5	variable	2.14 25.6	3.95	1.04 108.1	3.13	1.13 83.4	3.15	0.90 146.0	8.04
T6	variable	1.00 29.6	4.64	0.93 105.0	2.81	1.04 76.8	3.36	0.91 140.4	8.09
T7	variable	0.91 29.0	3.73	1.15 100.5	2.72	1.05 76.0	3.00	0.93 138.9	8.19
Spring	Section 10 (°)	11 (mm)	12 (mm) (°)	13 (mm)	14 (mm) (°)	15 (mm)	16 (mm) (°)	17 (mm)	
T3	0.85 145.0	3.28	1.49 75.4	3.14	1.03 110.2	3.84	3.02 28.0	variable	
T4	0.90 141.8	3.49	1.53 79.9	3.09	0.89 103.2	4.04	2.25 29.5	variable	
T5	0.89 142.2	3.31	1.53 76.0	3.14	0.99 106.6	4.37	1.61 28.0	variable	
T6	0.93 143.1	3.51	1.53 79.9	3.40	1.01 104.4	3.69	2.18 26.4	variable	
T7	0.84 144.0	3.51	1.68 79.5	3.21	1.29 108.0	3.35	3.31 23.3	variable	

Table 3.2: Repeatability Test: Archial-Loop Spring Geometries

Spring	Section 1 (mm) (°)	2 (mm) (°)	3 (mm)	4 (mm) (°)	5 (mm)	6 (mm) (°)	7 (mm) (°)
A3	16.48 variable	1.08 487.6	5.43	1.03 154.4	5.70	0.96 488.1	19.71 variable
A4	16.11 variable	1.03 489.5	5.88	0.94 157.4	5.71	1.08 486.5	16.79 variable
A5	14.49 variable	1.01 481.6	5.33	0.88 153.4	5.92	1.01 475.6	15.09 variable
A6	15.11 variable	1.03 480.3	5.79	0.88 154.2	5.62	1.10 489.6	16.04 variable
A7	16.11 variable	1.04 483.1	5.80	0.92 154.5	5.24	1.09 482.0	16.88 variable

Table 3.3: T-Loop Spring Geometries: Planar and Non-Planar Tests

Section	Straight or Curved	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Straight	T1	variable	-	-
		T2	variable	-	-
2	Curved	T1	-	2.13	30.1
		T2	-	2.12	30.2
3	Straight	T1	4.26	-	-
		T2	4.12	-	-
4	Curved	T1	-	1.15	98.9
		T2	-	1.04	104.9
5	Straight	T1	2.97	-	-
		T2	2.84	-	-
6	Curved	T1	-	1.06	88.6
		T2	-	1.11	80.5
7	Straight	T1	2.94	-	-
		T2	2.70	-	-
8	Curved	T1	-	1.11	145.3
		T2	-	1.00	143.9
9	Straight	T1	7.43	-	-
		T2	8.23	-	-
10	Curved	T1	-	0.98	143.2
		T2	-	0.80	139.8
11	Straight	T1	3.21	-	-
		T2	3.69	-	-
12	Curved	T1	-	1.03	85.2
		T2	-	1.20	79.4
13	Straight	T1	3.06	-	-
		T2	3.26	-	-
14	Curved	T1	-	1.08	102.7
		T2	-	1.02	105.9
15	Straight	T1	3.92	-	-
		T2	3.47	-	-
16	Curved	T1	-	3.83	24.2
		T2	-	3.20	24.1
17	Straight	T1	variable	-	-
		T2	variable	-	-

Table 3.4: Vertical-Loop Spring Geometries: Planar and Non-Planar Tests

Section	Straight or Curved	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Straight	V1	variable	-	-
		V2	variable	-	-
2	Curved	V1	-	1.16	514.1
		V2	-	1.08	514.8
3	Straight	V1	4.08	-	-
		V2	4.23	-	-
4	Curved	V1	-	1.07	164.5
		V2	-	1.04	165.8
5	Straight	V1	4.21	-	-
		V2	3.96	-	-
6	Curved	V1	-	1.15	515.8
		V2	-	1.14	514.3
7	Straight	V1	variable	-	-
		V2	variable	-	-

Table 3.5: Archial-Loop Spring Geometries: Planar and Non-Planar Tests

Section	Straight or Curved	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Curved	A1	-	14.30	variable
		A2	-	14.72	variable
2	Curved	A1	-	1.06	481.3
		A2	-	1.01	473.3
3	Straight	A1	5.59	-	-
		A2	5.60	-	-
4	Curved	A1	-	1.11	165.0
		A2	-	1.12	161.0
5	Straight	A1	5.01	-	-
		A2	5.10	-	-
6	Curved	A1	-	1.04	482.6
		A2	-	1.04	476.9
7	Curved	A1	-	14.81	variable
		A2	-	15.16	variable

A comparison between the actual and digitized geometry of the spring types is shown in Figure 3.3. In this figure, the digitized springs are overlaid on a the digital scan of the experimental springs as means of showing the error between the digitized and experimental geometry. It should be noted that, while only the center line of the digitized spring is shown, the actual width of the digitized wire is that of the empirical springs. The differences between the actual springs and the numerically digitized springs can be attributed to both measurement error and inaccurate modeling.

Measurement error is present due to both the limitations of the measurement equipment and the measurement procedure itself. Error from the procedure is present because the measurement of each spring involved visual inspection to determine at what point the curved sections becomes a straight sections and vice versa. Magnification was used during the measurement process to mitigate the error resulting from visual inspection. Error from the measurement equipment is present because the resolution of the measurement system was ± 0.02 mm. Repeated measurements were also conducted on each spring to confirm that the measured lengths and angles and the calculated radii were repeatable. This resulted in a standard deviation of 0.07 mm for the length measurements and 0.2° for angle measurements. Since the radii of the curved sections were calculated using $R = \frac{1}{8h}(c^2 + 4h^2)$ (see Figure 2.4 and Appendix B) the error in the radii varied depending on the values for h and c . The largest error in the radii was found for sections 2 and 16 of the T-springs. Error for these radii was determined to be as large as 0.7 mm. The reason for this larger measurement error is that the value for h was near the resolution of ± 0.02 mm.

Modeling error is also present because all curved sections were assumed to have constant radii when in fact it some variation exists. For example, the radii of the helical sections (2 and 6) of the archial-loop and vertical-loop springs were found to vary by as much as 0.16 mm depending on where the measurement was taken. Therefore, the error resulting from the constant radius modeling of these sections is larger than the actual measurement error.

The variability between the various clinically identical springs shown in Tables 3.1 and 3.2 can be seen by comparing the lengths, radii, and angles of each individual section. For example, the maximum difference in angle of section 2 amongst the 5 repeatability archial-loop springs is 9.2 degrees. With regards to the 5 repeatability T-springs, there is as much as 1 mm difference amongst the lengths of section 15. It should be noted, however, that differences amongst the springs of similar design are expected because of the manually bending of these springs. The effects of this variability will be further discussed in Section 3.3 of this chapter.

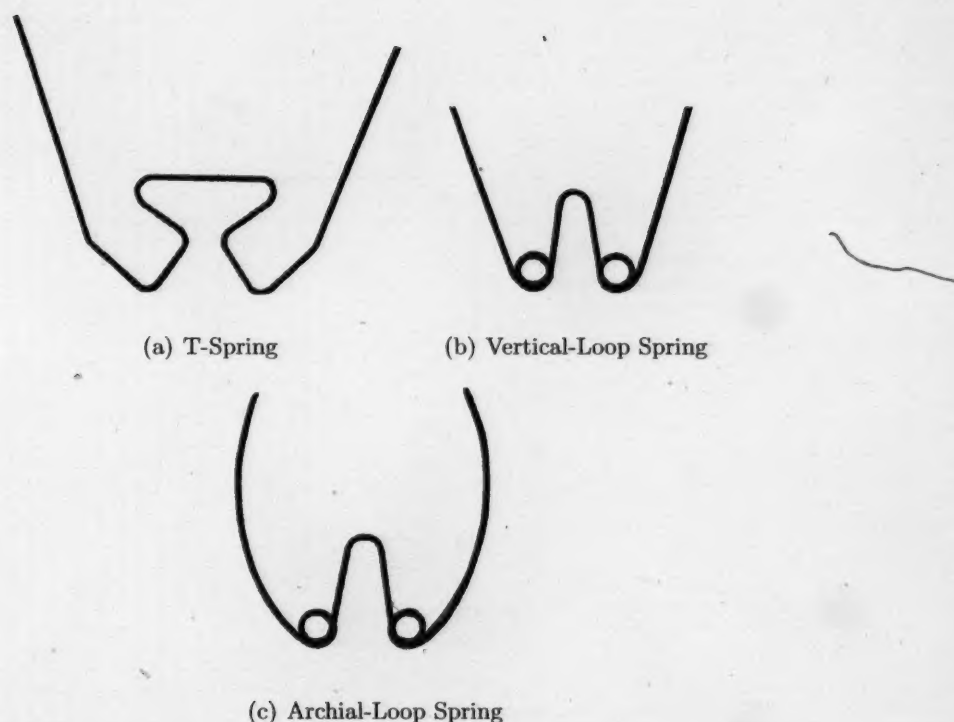


Figure 3.3: Digitized (Black) vs. Actual Geometries (Grey)

3.3 Spring Repeatability Study

3.3.1 T-Springs

Table 3.1 shows the measured geometries of the 5 T-springs used in this repeatability study. It is important to note that when making these springs, the orthodontist was attempting to bend 5 identical springs. The variability in the geometry is due to the fact that the springs are bent manually.

The goal of this portion of this study is to gain an understanding of the variability in spring properties of similar springs bent by an orthodontist. The test which involved 5.0 mm lengths for sections 1 and 17 was selected for illustrative purposes. Results of tests with other lengths for these sections can be found in Appendix A.

Figure 3.4 shows the numerical and empirical results of the tensile force versus inter-bracket distance (IBD) for the 5 T-springs. It should be noted that

force² versus IBD is used instead of force versus activation because such results are more clinically relevant. This is because the IBD is known clinically and the activation is not. The true activation of a spring at a particular inter-bracket distance can be found using *Activation = IBD - Neutral Position Length*.

The slopes of these force-IBD curves range from 0.40 N/mm to 0.44 N/mm numerically and 0.44 N/mm to 0.48 N/mm empirically. This indicates strong consistency amongst the springs. However, Figure 3.4 does show that there is variation in the level of force generated by the springs for a given IBD. The variation in force for a given IBD is as great as ~ 0.4 N (numerically) and ~ 0.6 N (empirically). The reason for this variation is the difference in neutral positions of each of the springs. For example, for the results shown in Figure 3.4(a), the neutral position of T-spring 3 (T3) was 21.3 mm, whereas it was 20.4 mm for T-spring 4 (T4). Therefore, for any given IBD, the activation of T4 is higher than that of T3. The same can be said for the empirical results shown in Figure 3.4(b). While the neutral positions were not measured, from the slopes of the lines it was determined that the neutral position of T3 was 20.4 mm and T4 was 21.3 mm. One likely reason for discrepancy between the numerical and empirical force versus IBD relationship of each spring is error in the lengths of section 1 and 17. That is, since the lengths of sections 1 and 17 are measured in-place with a caliper and a ruler in the experimental apparatus, there is larger uncertainty (± 0.5 mm) in these lengths than the lengths or radii of the other sections. Because of this error, it is possible that there were discrepancies between the numerical and experimental lengths of sections 1 and 17. For example, by adding 0.5 mm (a clinically small amount) to the lengths of sections 1 and 17 in the digital model, the neutral position of a spring is changed by ~ 1.0 mm (it is not a direct linear relationship). This causes a shift along the IBD axis of ~ 1.0 mm and results in a decrease in force of ~ 0.45 N for the T-springs shown. Such a decrease would increase agreement between the numerical and empirical force values for T5 and T6. Similar adjustments could also be made for T3, T4, and T7. Further analysis regarding the uncertainty in the spring arm length is presented in Section 3.4.1.

Figure 3.5 shows the moments³ about the *y*-axis for the 5 T-springs. Only the moments for the left end of the spring are shown because similar results were documented for the right end. A maximum difference of ~ 1.8 Nmm was found numerically for a given IBD. This increased to a maximum difference of

²While g has traditional been used in orthodontics as the unit for force, the unit of N is used in this work. ($1000 \text{ g} = 1 \text{ N} / 9.81 \frac{\text{m}}{\text{s}^2}$)

³While gmm has traditional been used in orthodontics as the unit for moment (torque), the unit of Nmm is used in this work. ($1000 \text{ gmm} = 1 \text{ Nmm} / 9.81 \frac{\text{m}}{\text{s}^2}$)

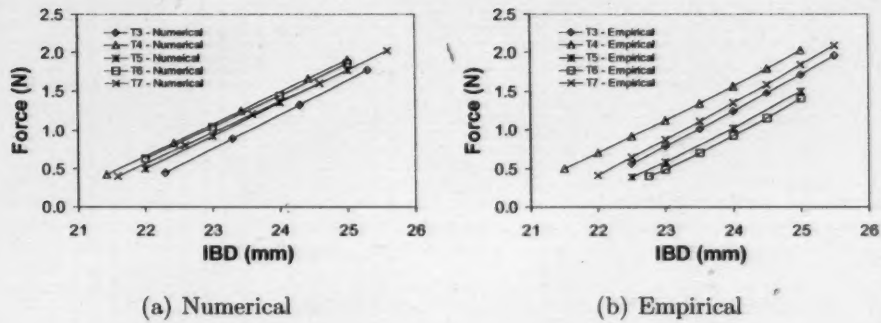


Figure 3.4: T-Spring Repeatability - F_z (Tension)

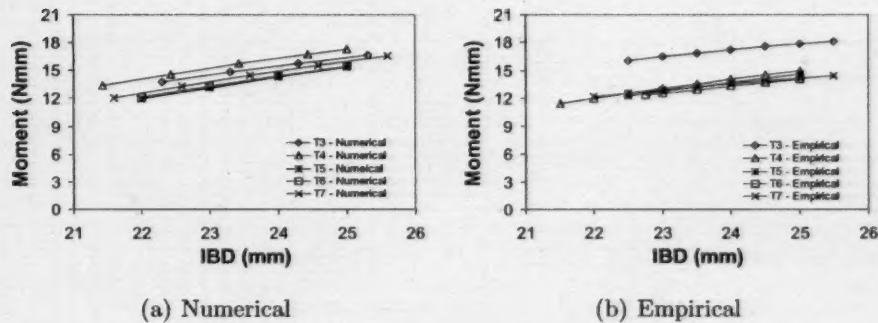
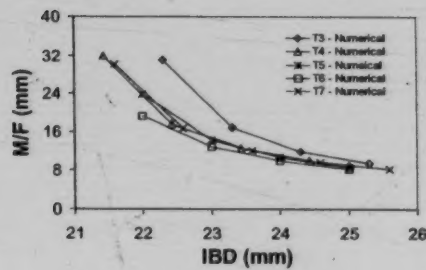


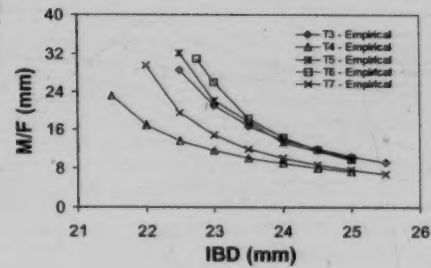
Figure 3.5: T-Spring Repeatability - M_y

~ 3.7 Nmm empirically. Again, one likely reason these discrepancies is error in the lengths of the spring arms (sections 1 and 17). This is because spring arm length significantly affects the applied moment as well as the neutral position length. This will be further validated in Section 3.4.1.

Of particular interest to the clinical orthodontist is the values of the moment-to-force ratios at specific inter-bracket distances. Figure 3.6 shows the M/F ratios for this test with the specific values in Table 3.6. These plots show that at an IBD of 25 mm the M/F ratio values range from 8.3 mm to 10.5 mm numerically and 7.4 mm to 10.4 mm empirically. This is within 24% and 22%, respectively, of the 8.5 mm that reportedly produces translation. While these differences are clinically significant (i.e. M/F ratios of 7.9 mm and 9.5 mm can lead to controlled tipping and root movement respectively for a central



(a) Numerical



(b) Empirical

Figure 3.6: T-Spring Repeatability - M/F Ratios

Table 3.6: T-Spring Repeatability Study M/F Ratios

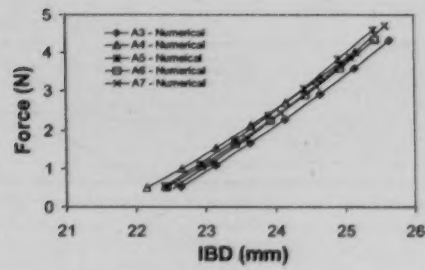
M/F Ratio (mm)									
IBD (mm)									
T3		T4		T5		T6		T7	
Num	Emp	Num	Emp	Num	Emp	Num	Emp	Num	Emp
-	-	32.0	23.0	-	-	-	-	30.2	-
21.5	21.5	21.4	21.5	21.5	21.5	21.5	21.5	21.6	21.5
-	-	-	17.0	23.9	-	19.3	-	-	29.5
22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0
31.1	28.6	17.5	13.7	-	32.1	-	30.9	16.6	19.5
22.3	22.5	22.4	22.5	22.5	22.5	22.5	22.8	22.6	22.5
-	21.0	-	11.6	14.3	22.1	12.9	26.0	-	14.8
23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0
16.8	16.7	12.9	10.1	-	-	-	18.4	12.1	12.0
23.3	23.5	23.4	23.5	23.5	23.5	23.5	23.5	23.6	23.5
-	13.9	-	9.0	10.6	13.4	10.0	14.3	-	10.0
24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0
11.9	11.9	10.0	8.1	-	-	-	11.8	9.7	8.7
24.3	24.5	24.4	24.5	24.5	24.5	24.5	24.5	24.6	24.5
-	10.4	9.1	7.4	8.6	9.8	8.3	10.0	-	7.7
25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
9.4	9.3	-	-	-	-	-	-	8.2	6.9
25.3	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.6	25.5

incisor (Tanne, K et al., 1988)), Figures 3.6(a) and 3.6(b) show that each of the springs would likely be able to produce a M/F ratio of ~ 8.5 mm if it was adjusted properly in the mouth. Furthermore, while this study assumes that a M/F ratio ~ 8.5 mm leads to translation, previous works have used values of ~ 9.4 mm (Siatkowski, RE, 1994) and 14 mm (Nikolai, J, 1985); thus, indicating that higher M/F ratios may be needed. As the IBD decreases it is shown that the M/F ratio increases. To clinically compensate for this an orthodontist shortens the ends of sections 1 and 17 accordingly. This is because shortening these lengths decreases the neutral position of the spring. Correspondingly, this increases the activation of the spring. Therefore, for a given IBD the force applied to the tooth is increased. This is similar with regards to the applied moments in that the shorter ends also result in a higher moment on the tooth. The net result, however, is generally a decrease in M/F ratio closer to 8.5 mm. It should be noted that, while the M/F ratio values for T6 are significantly higher empirically than numerically, this is due to the fact that the force values for T6 were significantly lower empirically than numerically. As previously discussed, it is likely that this is due to error in the lengths of sections 1 and 17.

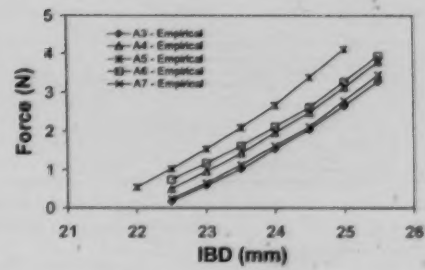
3.3.2 Archial-Loop Springs

Table 3.2 shows the measured geometries of the 5 archial-loop springs used in this repeatability study. Again, it is important to note that when making these springs the orthodontists was attempting to bend 5 identical springs. The variability in the geometry is due to the fact that the springs are bent manually. It should also be noted that all curved sections are assumed to be of a constant radius even though this may not be entirely correct.

Figure 3.7 shows the numerical and empirical results of the tensile force for the 5 A-Springs. The test with bisection (illustrated in Figure 3.8) lengths of 9 mm for sections 1 and 7 was selected. Results from other tests can be found in Appendix A. The bisection measurement was selected because it is difficult to experimentally measure arclength. Arclength was subsequently calculating using the bisection measurement and the known radius of the section. It should again be noted that error results because of the uncertainty in the lengths and angles of sections 1 and 7 when placing in the testing equipment. Similar to the T-springs, the shifts along the IBD axis for each spring are representative of the different neutral positions of each spring. These shifts result in a maximum difference of ~ 0.5 N (numerically) and ~ 1.5 N (empirically) for a given IBD. Figure 3.9 shows the moments about the y -axis for the 5 A-springs. Again,



(a) Numerical



(b) Empirical

Figure 3.7: Archial-Spring Repeatability - F_z (Tension)

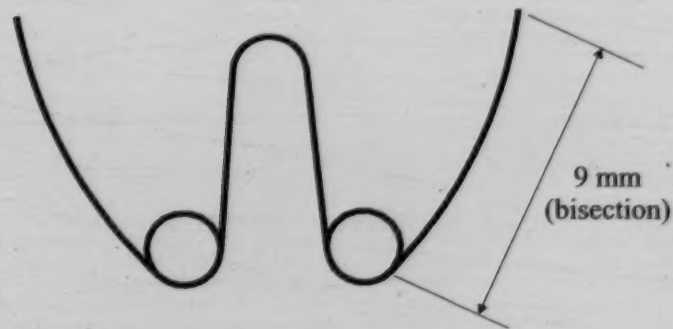
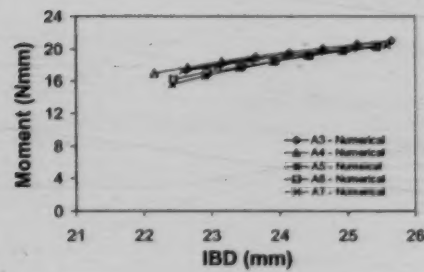
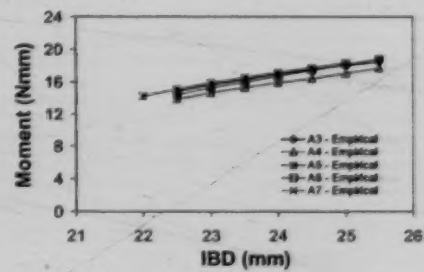


Figure 3.8: Archial Spring Section 1 and 7 Bisection Length



(a) Numerical



(b) Empirical

Figure 3.9: Archial-Spring Repeatability - M_y

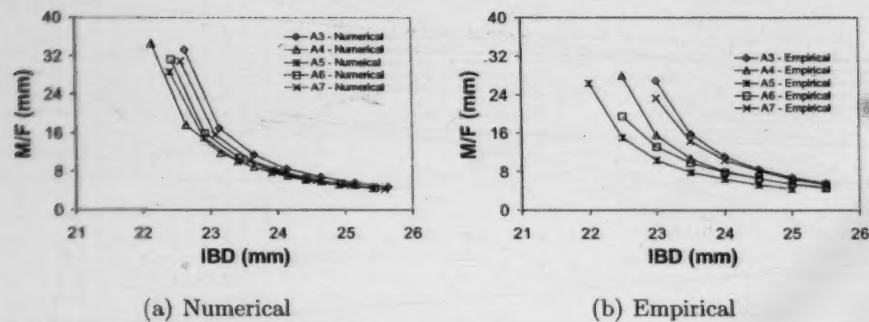


Figure 3.10: Archial-Spring Repeatability - M/F Ratios

only the moments for the left end of the spring are shown because similar results were documented for the right end. A maximum difference of 1.7 Nmm was found numerically while a maximum difference of 1.3 Nmm was found experimentally. While both of these maximum differences are $\sim 11\%$ of the applied moment, it is the resulting variation in the M/F ratio that is of most concern clinically.

The moment-to-force ratios for the 5 A-springs are shown in Figure 3.10 with specific values in Table 3.7. These plots show that at an IBD of 25 mm the M/F ratio values range from 5.2 mm to 6.2 mm numerically and 4.5 mm to 6.8 mm empirically. All of these are below the desired 8.5 mm value for translation. However, this does not indicate that the spring is poorly designed. To increase these M/F ratios the arclengths of section 1 and 7 could be increased. This would result in an increase in the length of the neutral position of the spring, and thus, would lower the force for a given IBD. This in turn, would increase the M/F ratio for that IBD. As the intention of this section is to investigate the variability amongst clinically identical springs, the "fitting" of a specific M/F ratio to a specific IBD was not investigated.

3.4 Planar Results

As illustrated in Section 3.3, variability exists between identical springs. Therefore, prior to the full three-dimensional analysis, planar tests were conducted to determine sources of error and to validate the testing technique. The following sections show the results from these planar tests. Only one T-loop spring, one vertical-loop spring, and one archial-loop spring were investigated for these

Table 3.7: A-Spring Repeatability Study M/F Ratios

M/F Ratio (mm)									
IBD (mm)									
A3		A4		A5		A6		A7	
Num	Emp	Num	Emp	Num	Emp	Num	Emp	Num	Emp
-	-	34.5	-	-	26.3	-	-	-	-
22.0	22.0	22.1	22.0	22.0	22.0	22.0	22.0	22.0	22.0
33.3	-	17.6	27.8	28.7	15.0	31.3	19.5	30.9	-
22.6	22.5	22.6	22.5	22.4	22.5	22.4	22.8	22.6	22.5
16.9	26.9	11.9	15.5	14.9	10.4	16.0	13.2	15.6	23.2
23.1	23.0	23.1	23.0	22.9	23.0	22.9	23.0	23.1	23.0
11.4	15.9	9.0	10.7	10.2	7.9	10.8	9.9	10.5	14.4
23.6	23.5	23.6	23.5	23.4	23.5	23.4	23.5	23.6	23.5
8.6	11.2	7.2	8.1	7.7	6.4	8.2	7.9	7.6	10.4
24.1	24.0	24.1	24.0	23.9	24.0	23.9	24.0	24.1	24.0
6.9	8.5	6.0	6.6	6.2	5.2	6.6	6.7	6.3	8.3
24.6	24.5	24.6	24.5	24.4	24.5	24.4	24.5	24.6	24.5
5.7	6.8	5.1	5.4	5.2	4.5	5.5	5.5	5.2	6.5
25.1	25.0	25.1	25.0	24.9	25.0	24.9	25.0	25.1	25.0
4.9	5.6	-	4.6	4.4	-	4.7	4.8	4.4	5.5
25.6	25.5	25.6	25.5	25.4	25.5	25.4	25.5	25.6	25.5

planar tests. The springs tested were T1, V1, and A1 from Tables 3.3, 3.4, and 3.5 respectively. Each of these springs was tested 3 times in the planar configurations. This involved removing the spring from the apparatus and realigning the apparatus before repeating the test a second and third time.

3.4.1 T-Loop Springs

Figure 3.11 shows the results of one of the planar (only z activation) tests for the T-spring (T1) tested. Results of the other tests are not shown because they show very similar results. These results are included in Appendix A. Activation is used instead of inter-bracket distance because this is the more common approach used in previous works. Furthermore, activation is more applicable for an engineering analysis because zero activation corresponds to zero force. For this specific test, the lengths of sections 1 and 17 were each taken as 6.0 mm. To get this length, the spring arm was initially slide through

the mounting bracket until an arm length of 6.0 mm was reached. This length was measured with a caliper and a ruler. The excess wire was then bent around the bracket. This process could only be completed for the first of the three tests however. This is because when removing the spring from the mounting bracket, the end wire which was bent around the mounting bracket had to be straightened to slide back out of the bracket. Because of the bend, the wire could not totally be straightened. The plastic deformation resulted in a small "kink" left in the wire. This meant that when the wire was reinserted for the second and third tests and re-bent around the mounting bracket that the bend would naturally occur in the same location as that of the first test. It was possible to shorten the arm lengths by ~ 2 mm and make a new bend but not to make a new bend at the same arm length (in this case 6.0 mm). While this may have resulted in repeated error over the three tests, it did result in consistency in the lengths of section 1 and 17 for the three tests.

Figure 3.11(a) shows a plot of the spring's axial force and Figure 3.11(b) shows the shear force in the x direction. Because this is a planar test, no shear force in the y direction is expected. This was confirmed as F_y was measured to be essentially zero (< 0.02 N). Figures 3.11(c) and 3.11(d) show the results of the measured and simulated moments for the planar tests. Again, out-of-plane moments are not shown because they were predicted to be zero and measured to be small (< 1 Nmm). Each data point in the experimental results is an average of 10 measurements. However, no error bars are shown on the force curves because the standard deviation and variance of the 10 data points were determined to be smaller than the scale of the figure. For example, the standard deviation and variance of F_z for the test shown in Figure 3.11 are 4.7×10^{-4} N and 2.2×10^{-7} N² respectively. Such a result is consistent with all force and moment measurements presented in this work and therefore, error bars are not shown in subsequent figures. The left and right labels in Figure 3.11 indicate which end of the spring the data is associated with (see Figure 3.2(a) and Table 3.3). No numerical left or right is denoted on the axial force curve plot because both are predicted to be positive and of the same magnitude.

Also included on the plot is the corrected left and right curves. The corrected plots represent the empirical curves with a shift along the activation axis. This activation shift is required because for this T-spring design the neutral position could not be found experimentally. This is because, when applying the required moments and shear forces to each end of the spring to achieve the neutral position, contact between the vertical sections of the spring (sections 6 and 12) occurred before the neutral position could be reached. Figure 3.12(a) demonstrates this. This contact resulted in the inability of the

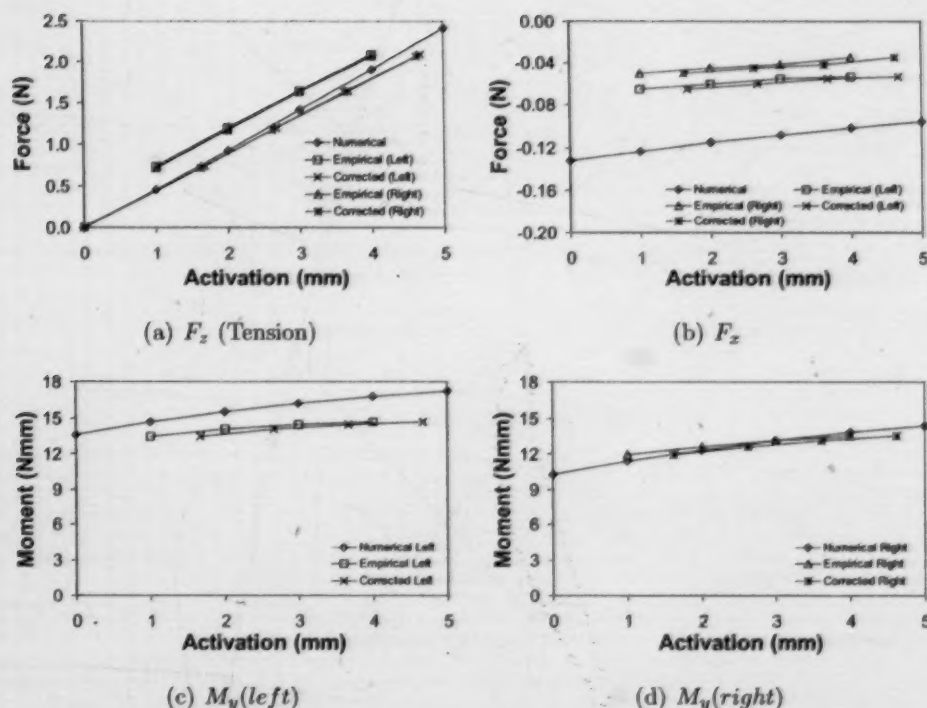


Figure 3.11: T-Spring Force / Moment Results (Planar Tests)

spring to deform to its true neutral position (shown in Figure 3.12(b)). Therefore, no point of zero axial force and zero activation could be reached. Since activation is measured from the spring's true neutral position, it was not possible to determine the activation experimentally. To determine the activation, the empirical tensile force-activation data was initially plotted using the point of contact as zero activation. This resulted in the *Empirical* curves shown in Figure 3.11(a). The correction factor shift was then determined by extrapolating the empirical curve to determine where the curve crosses the activation-axis and shifting the curve along the activation-axis so that the curve intercepts the activation and force axes at (0, 0). This activation shift results in the *Corrected* curve. For the experimental tests plotted the activation shift was found to be 0.67 mm for the test with the left end attached to the load cell and 0.62 mm for the test with the right end attached to the load cell. This discrepancy of 0.05 mm can likely be attributed to uncertainties in the placement of the spring in the brackets. Using these activation shifts, the experimentally

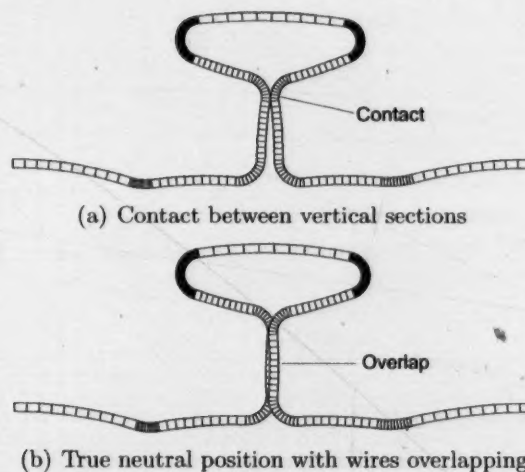


Figure 3.12: Wire Contact Affecting Neutral Position

measured neutral position of the T1 was determined to be 24.5 mm. This is 0.8 mm lower than the numerically predicted value of 25.3 mm. Furthermore, with this shift the numerical model predicts the tensile force within 4% of the experimental value. The spring constant was determined to be 0.48 N/mm numerically compared to a measured value of 0.46 N/mm. It is important to note that this activation shift, calculated from the force-activation curve, is also used for all other force and moment results pertaining to T1.

The force in the x -direction (in-plane shear force), which is shown in Figure 3.11(b), is due to the fact that the spring being tested was a non-symmetric (right side does not mirror the left side) spring. Although the numerical prediction is approximately twice that of the empirical, the values are small and would not be clinically relevant. The discrepancies can likely be attributed to non-ideal boundary conditions, small misalignments in the testing apparatus, non-symmetries in spring placement in brackets, and digitization errors (non-straight sections of the spring or non-constant radii). The difference between the *empirical left* and *right* curves of approximately 0.02 N is likely a result of the spring being remounted in the brackets (once for the left end test and once for the right end test). In theory, these forces should be equal to maintain equilibrium. The force in the y direction (out-of-plane shear force) was measured to be essentially zero (< 0.01 N) for all planar tests. This is expected for a planar spring in a planar test. Furthermore, these small deviations from zero are clinically insignificant and likely due to small misalignments in the testing

apparatus, non-planar components of the spring, and measurement error.

The moments at both the left and right ends of the spring are shown in Figures 3.11(c), 3.11(d). Only M_y is shown because both M_x and M_z were found to be small (< 1 Nmm) experimentally and predicted to be zero numerically. Again, the deviations from the numerically predicted value of zero for M_x and M_z can be attributed to non-ideal boundary conditions, initial geometry assumptions, and small misalignments in the testing apparatus. Of particular interest with regards to M_y is that different moments were applied at the *Left* and *Right* ends. A difference is expected because of the asymmetric spring but the numerical difference of ~ 3 Nmm may be clinically significant if the intention was for a symmetric spring. This difference was found to be smaller experimentally however.

With regards to M_y , the numerical and empirical results shown were found to agree within 15%. However, similar to F_x , this discrepancy can be partially attributed to the likelihood of error resulting from asymmetries in spring placement in the mounting brackets. These asymmetries are partially a result of the lengths of sections 1 and 17. As mentioned in section 3.3, these lengths, which have a large effect on the moment produced, are the ones with the most uncertainty. This uncertainty (± 0.5 mm) is because they were measured with a caliper and ruler in-place. Therefore, this uncertainty is larger than that of the other section lengths or radii that were measured with a digital scan. It was not possible, however, to measure the spring arm lengths using a digital scan while the spring was mounted in-place. Furthermore, no magnification was available to determine where the section changed from straight to curved. This resulted in more-difficult visual inspection to determine where the straight 1 and 17 sections turned into the curved 2 and 16 sections respectively.

The effect that error in the lengths of these sections can have on these results is illustrated by the effects of adjusting the lengths of sections 1 (left) and 17 (right) in the numerical tests. The lengths of section 1 and 17 were changed from 6.0 mm to 7.0 mm and 6.7 mm respectively. The results of this change are shown in Figure 3.13. When comparing these results to the results shown in Figure 3.11, it is clear that this adjustment of the end lengths brings the numerical predictions into much closer agreement with the actual forces and moments measured. Both F_x and M_y are modeled more accurately while there is insignificant change in F_z . Changing the lengths of sections 1 and 17 does affect the neutral position length of the spring however. Numerically, such a change increased the neutral position from 25.3 mm to 27.1 mm. This could not be confirmed experimentally because the true neutral position could not be measured.

Since both lengths were increased to improve the numerical predictions,

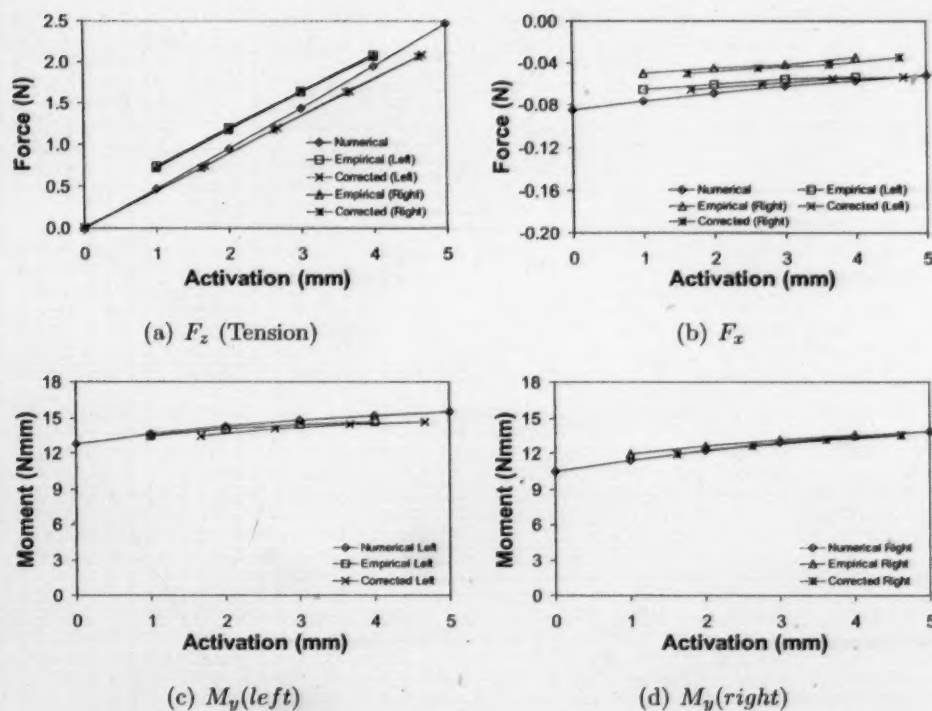
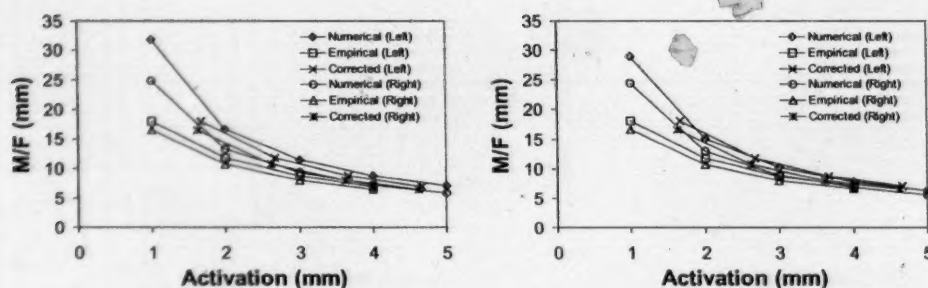


Figure 3.13: T-Spring Force / Moment Results (Planar Tests) with Sections 1 and 17 Correction

this may be evidence that the method used to measure these lengths in-place, consistently underestimates the true lengths. It may also indicate that the method used to hold the wire in the mounting brackets (bending the excess around the bracket) had some "give" in it and resulted in the lengths lengthening as the load was applied. Conversely, the lengthening of the end sections may be indicative of the non-ideal nature of the boundary conditions and that, instead of the end section lengthening due to a give in the bend, the end sections may be longer than that which was measured because the "true" end of the section is not at the edge of the bracket. That is, since the wire is not a perfect fit in the bracket (the bracket cross-section is 0.457×0.635 mm or $0.018'' \times 0.025''$), the section may continue into the bracket. This too would result in the net effect of the effective length being longer than that which was measured.

As mentioned in Chapter 1, of particular importance is the moment-to-force



(a) M/F Ratio (No end length correction) (b) M/F Ratio (End length correction)

Figure 3.14: T-Spring M/F Ratio

(M/F) ratio. This is because M/F ratio affects the type of tooth movement such as translation or tipping. Figure 3.14(a) shows the M/F ratios for the spring tested with the specific values listed in Table 3.8. Figure 3.14(b) shows the M/F ratio for the spring with the lengths of sections 1 and 17 changed from 6.0 mm to 7.0 mm and 6.7 mm respectively. Table 3.9 shows the specific M/F ratio values. From these two plots, it is clear that the small errors in the numerically predicted force and moment values do not significantly affect the moment-to-force ratio. The numerically predicted M/F ratio agree very well with the measured values at higher activations.

Table 3.8: T-Spring Planar Study M/F Ratios

Activation (mm)	M/F Ratio (mm)					
	Num. L	Num. R	Emp. L	Emp. R	Cor. L	Cor. R
1.0	31.9	24.8	18.1	16.6	-	-
1.6	-	-	-	-	18.1	16.6
2.0	16.7	13.3	11.7	10.8	-	-
2.6	-	-	-	-	11.7	10.8
3.0	11.5	9.3	8.8	8.1	-	-
3.6	-	-	-	-	8.8	8.1
4.0	8.8	7.3	7.0	6.6	-	-
4.6	-	-	-	-	7.0	6.6
5.0	7.2	6.0	-	-	-	-

Table 3.9: T-Spring Planar Study M/F Ratios (End Length Correction)

Activation (mm)	M/F Ratio (mm)	
	Num. L	Num. R
1.0	29.0	24.4
2.0	15.0	12.9
3.0	10.3	9.0
4.0	7.8	6.9
5.0	6.3	5.6

3.4.2 Vertical-Loop Springs

Figure 3.15 shows the results of one planar test for the vertical-loop springs (V1) tested. Results of the other tests are presented in Appendix A. Figure 3.15(a) shows the relationship between axial force and z activation. Comparison shows that the numerical model over-predicts the empirical tension by $\sim 27\%$ (1.9 N/mm vs. 1.5 N/mm). This is significantly more than the 4% over-prediction for the T-spring. One likely source of this error is the modeling of the helical sections. This is because these sections are modeled as planar elements with constant radii when in fact these helices have a small out-of-plane component and are not of constant radii. Furthermore, as the helices "wrap around", the wire contacts the wire which it is next to. This results in contact and friction forces which likely affect numerical modeling. Modeling error may also result because the radii of these helical sections are not much greater than the width of the wire cross-section. The smaller radii may result in error from the thin beam theory applied by the numerical method.

Similar to the T-springs, the vertical-loop springs were asymmetric. This resulted in a shear force in the x direction as shown in figure 3.15(b). No plot of the shear force in the y direction is given for these planar tests because this force was predicted to be zero and measured to be less than 0.05 N.

Figures 3.15(c) and 3.15(d) show the results of the measured and predicted moments for the planar tests. In this case the numerical procedure over-predicts the empirical results by between 10% and 17%; however, similar to the T-spring results in Section 3.4.1, it is possible that the end section arm lengths were in fact longer than that of the measured 7.0 mm. Longer end lengths would have resulted in lower moments at each end. No investigation into this end length adjustment was conducted though. Also, no out-of-plane moments are shown because they were predicted to be zero. It should be noted that these out-of-plane moments were measured experimentally and found to

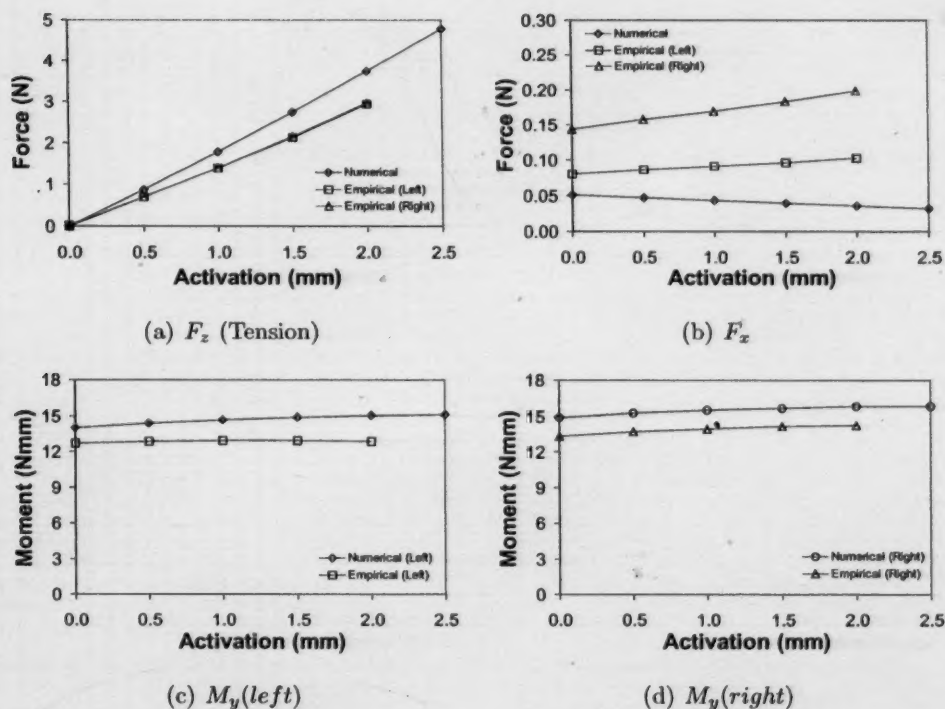


Figure 3.15: Vertical-Loop Spring Force / Moment Results (Planar Tests)

be less than 1.7 Nmm for M_x and less than 0.2 Nmm for M_z . The reason for these out-of-plane moments is the out-of-plane components of the helices which are not modeled numerically.

Of particular interest with regards to this test is the fact that the slope of the numerically predicted force in the x direction (Figure 3.15(b)) is opposite to that of the empirically measured force. This opposite slope can be explained using the concept of equilibrium in which the summation of moments must equal zero. For example, for a non-symmetric V-spring under planar activations, the forces and moments acting on the spring are as shown in Figure 3.16. When summing moments about the y -axis at the left end (neglecting the weight of the spring), the result is

$$+\circlearrowleft \sum M_y = 0 = M_{y(left)} - M_{y(right)} + F_{x(right)} \times IBD. \quad (3.1)$$

This shows that there is a direct relationship between F_x , $M_{y(left)}$, and $M_{y(right)}$

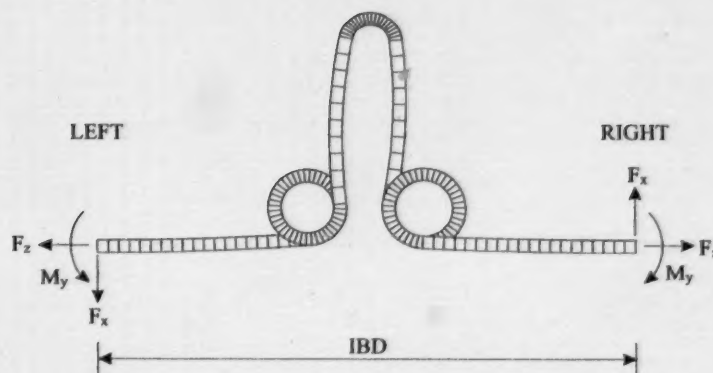


Figure 3.16: Vertical-Loop Spring Equilibrium

for a given IBD. When considering the results shown in Figure 3.15, it is clear that, for the empirical results presented, the difference between $M_{y(left)}$ and $M_{y(right)}$ is smaller at lower activations (lower IBD's) whereas for the numerical simulation it is smaller at larger activations (higher IBD's). This means that the $F_x \times IBD$ term must be smaller at lower activations empirically and smaller at larger activation numerically. Thus, this results in the different slopes shown in Figure 3.15(b).

The M/F ratios for the planar V-spring test are shown in Figure 3.17 with the values listed in Table 3.10. Again, it is shown that small errors in the numerical modeling of the forces and moments do not significantly affect the M/F ratio. In fact, even though there is a larger error in the modeling of the tensile force when compared to the T-spring (27% compared to 4%), there is less variability in the M/F ratio.

Unlike the T-Spring results discussed in Section 3.4.1, no correction factor is needed for the V1. This is typical for the vertical-loop spring design because no contact between spring sections occurred during deformation of the V-springs. As a result, the neutral position of V1 was directly measured to be 20.1 mm for the test shown. This is 1.6 mm higher than the 18.5 mm neutral position predicted numerically for the test shown. A possible cause of this under-prediction of the neutral position is the effect of the helices (sections 2 and 6) which are modeled as planar elements with constant radii when in fact these helices have a small out of plane component and do not have constant radii. Another possibility is that there was error in the lengths of the end sections (sections 1 and 7). For example, increasing the lengths of

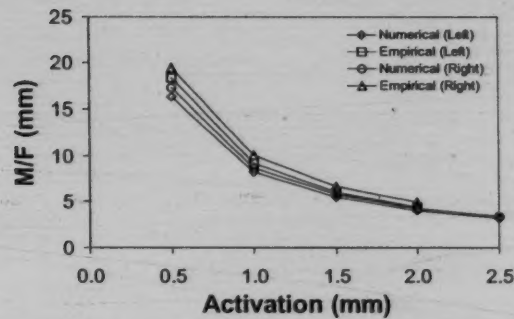


Figure 3.17: Vertical-Loop Spring M/F Ratio

Table 3.10: V-Spring Planar Study M/F Ratios

Activation (mm)	M/F Ratio (mm)			
	Num. L	Num. R	Emp. L	Emp. R
0.5	16.3	17.3	19.5	18.4
1.0	8.2	8.7	10.0	9.2
1.5	5.4	5.7	6.6	6.0
2.0	4.0	4.2	4.8	4.3
2.5	3.2	3.3	-	-

section 1 and 7 in the numerical simulation would increase the numerically predicted neutral position. Such lengthening would also result in a lowering of the numerically predicted moments. For example, increasing the lengths of sections 1 and 7 from 7.0 mm to 7.7 and 7.9 mm respectively, results in the predicted neutral position increasing from 18.5 to 20.0 mm. The resulting moments are 13.9 and 14.0 Nmm respectively.

3.4.3 Archial-Loop Springs

Figure 3.18 shows the results of the planar test for the archial-loop spring (A1) tested. Such a spring design is similar to the vertical-loop spring but with curved end sections. Figure 3.18(a) shows the relationship between axial force and axial activation. Like the vertical-loop spring analyzed in Section 3.4.2, the numerical model over-predicts the axial force. For the results shown in Figure 3.18(a) the over-prediction is 29%. This is very similar to the 27%

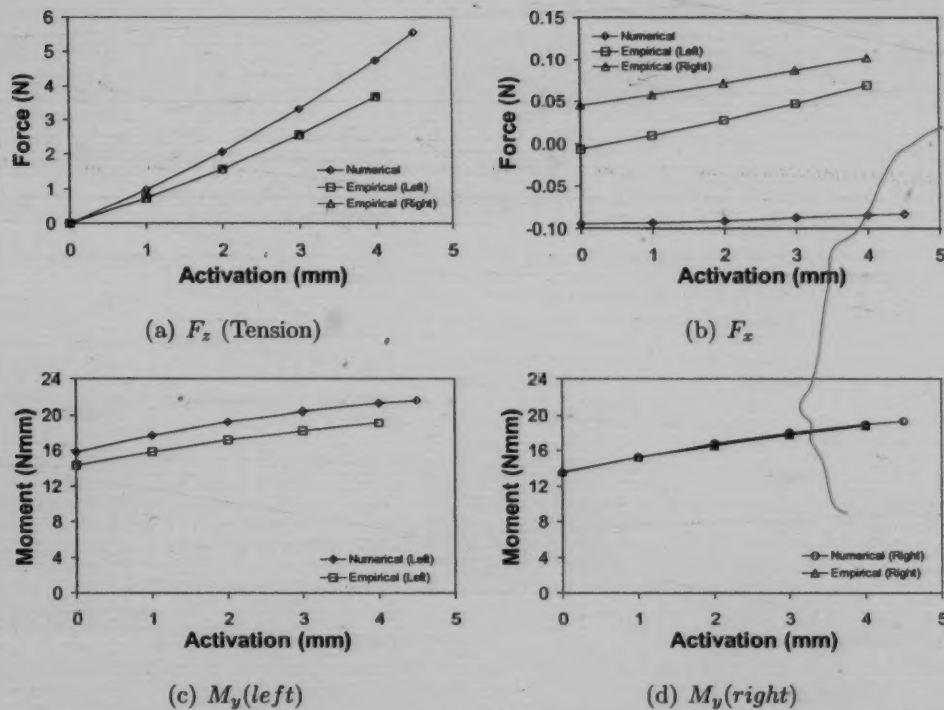


Figure 3.18: Archial-Loop Spring Force / Moment Results (Planar Tests)

over-prediction for the vertical-loop spring and likely indicates that there is common error between the numerical prediction of the tensile force for the V- and A-spring designs. Since an over prediction of 4% was seen for the T-spring, a logical conclusion is that the helices present in the vertical and archial-loop designs affect the modeling. With regards to A1's tensile force - activation relationship, it should also be noted that the numerical model predicts the slight non-linearity that was measured experimentally.

Similar to the T- and V-springs, the A-springs were asymmetric. This resulted in a shear force in the x direction as shown in figure 3.18(b). The difference in sign of the F_x values between the numerical and empirical tests can be explained in a similar manner to that of the opposing slope of F_x from the vertical-loop spring. That is, when summing the moment about the y -axis (see Equation 3.1) for this spring, it is clear that the difference between the numerical $M_{y(left)}$ and $M_{y(right)}$ is positive because $M_{y(left)}$ is larger. This is not the case empirically as shown in Figures 3.18(c) and 3.18(d). From this it

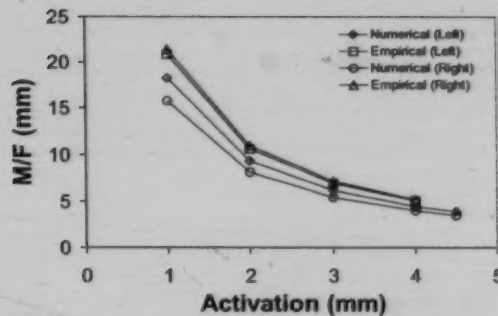


Figure 3.19: Archial-Loop Spring M/F Ratio

follows that F_x must be in the negative direction to maintain equilibrium.

Again, no plot of the shear force in the y direction is given for these planar tests because this force was predicted to be zero and measured to be small (< 0.17 N). It should be noted that this 0.17 N is larger than the 0.02 N and 0.05 N measured for the T-loop (Section 3.4.1) and vertical-loop (Section 3.4.2) springs respectively and that it is likely due to initial non-planar components of the spring and / or misalignments in the testing apparatus. No out-of-plane moments are shown because they were predicted to be zero and measured to be small (< 2 Nmm for M_x and < 1 Nmm for M_z). Similar to the V-spring, the neutral position was able to be found experimentally (24.5 mm numerically and 24.5 mm empirically for the test shown).

To improve the numerical results for $M_{y(left)}$ and F_x , end length adjustments of sections 1 and 7 could be conducted. While no investigation into this was conducted, it is likely that increasing the length of section 1 in the numerical model of A1 would decrease the moment at the left end. F_x would adjust accordingly as well. It should be noted, however, that increasing the length of section 1 would also increase the length of the neutral position. This would decrease agreement between the predicted and measured neutral positions.

Figure 3.19 shows the moment-to-force ratio for this archial-loop spring under planar activations. The specific values are listed in Table 3.11. For both the left and right ends the numerical simulations under-predicted the empirical results due to the significant over prediction of the axial force.

Another point of interest regarding the archial-loop spring design is the increased maximum activation compared to that of the vertical-loop spring. While V1 yielded at ~ 2.5 mm, A1 yielded at ~ 4.5 mm. This is an important

Table 3.11: A-Spring Planar Study M/F Ratios

Activation (mm)	M/F Ratio (mm)			
	Num. L	Num. R	Emp. L	Emp. R
1.0	18.3	15.8	20.9	21.4
2.0	9.3	8.1	10.6	10.9
3.0	6.1	5.4	6.9	7.1
4.0	4.5	4.0	5.1	5.2
4.5	3.9	3.5	-	-

characteristic clinically because it shows that, for the geometries considered, archial-loop springs can be used for larger space closures. The results shown in Figures 3.17 and 3.19 also indicate that there is an increased desirable activation range. That is, there is a larger range in which the M/F ratio is between 7 and 10 mm (~ 2 mm for the archial-loop spring and ~ 1 mm for the vertical-loop spring). This means there is a larger range in which the force system will lead to translation of the tooth ($M/F \approx 8.5$ mm). This is clinically beneficial.

3.5 Three-Dimensional Tests

Once it was confirmed that the experimental testing technique and numerical modeling procedure can be used to accurately measure and model the mechanical behavior of the orthodontic retraction appliances for 2D activations, where the mechanics is well understood and easier to visualize, 3D activations were investigated. For the complete three-dimensional tests, one end of the spring was displaced in combinations of x , y , and z displacements. Each three-dimensional test was repeated 3 times and 10 measurements were recorded at each activation. Again, no standard deviation is shown in the subsequent figures because, similar to the planar tests, it is smaller than the scale of the figures.

3.5.1 T-Loop Springs

To illustrate the experimental versus numerical results of a complete three dimensional force system for an orthodontic T-spring, the test involving an initial y displacement of 2 mm was selected. Figures 3.20(a)-(c) show the force results of the non-planar test of T-spring1 (T1) with 6.0 mm end sections. The

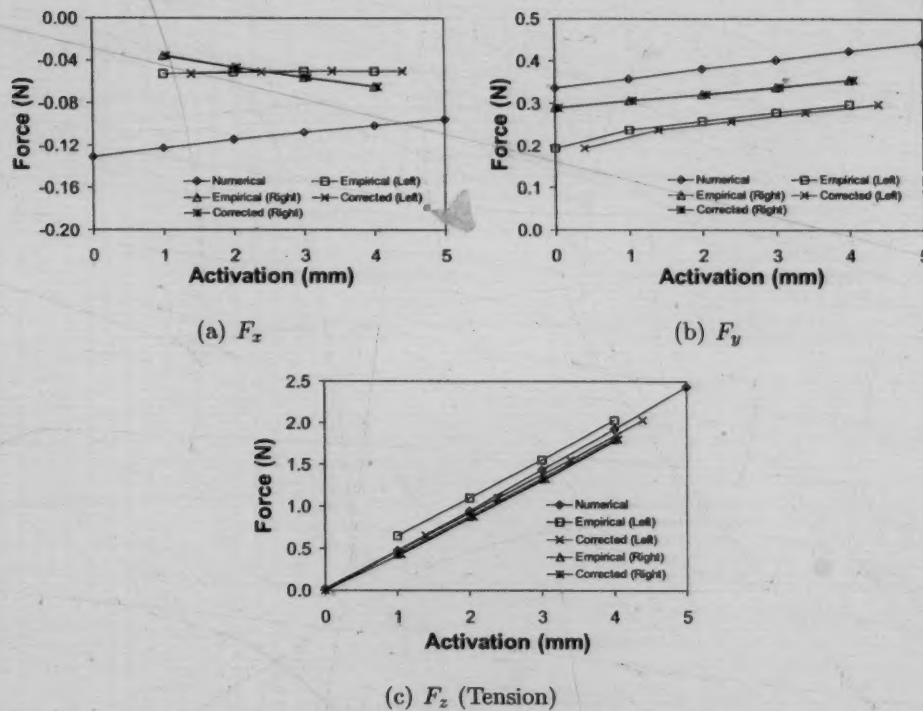


Figure 3.20: T-Spring Force Results ($y = 2$ mm)

measured axial (F_z) spring constants of 0.48 N/mm (numerical), 0.46 N/mm (empirical left), and 0.45 N/mm (empirical right) indicate that the y displacement does not affect the tensile force of the spring when compared to planar scenarios because the axial spring constants were insignificantly changed (i.e. 0.48 N/mm (numerical) and 0.46 N/mm (experimental) for the planar tests). This is an important fact for orthodontists because it means that unwanted out-of-plane displacements can occur and not affect the tensile forces. Similarly, F_x is not significantly affected by the y displacement. This is shown in Figure 3.21 which compares F_x for the planar and three-dimensional tests.

As expected, the introduction of a y displacement does affect forces in the y direction. A spring force perpendicular to the sagittal plane (shown in Figure 3.20(b)) results from a y displacement. Numerically, this out-of-plane force was found to vary linearly with z -activation with a slope of 21 mN/mm. This agrees with the experimentally determined slope of 20 mN/mm within 5%. Again, this discrepancy in the magnitude of the force is likely due to small

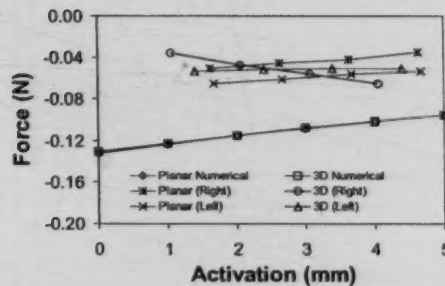
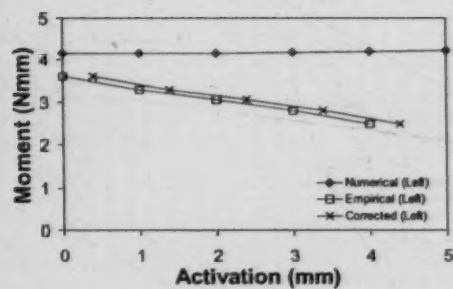


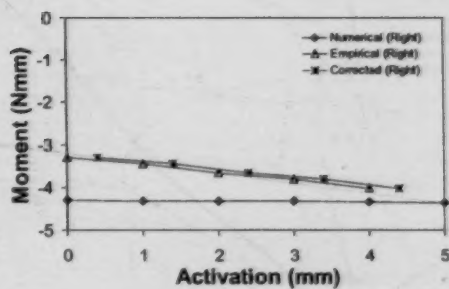
Figure 3.21: T-Spring F_x - Planar vs. Three-Dimensional Results

misalignments in the testing apparatus and/or errors in digitizing the spring.

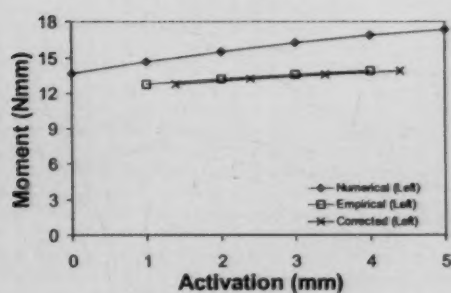
With regards to the moments created by the T-springs, the introduction of a y displacement introduced moments about both the x and z -axes. These results are shown in Figure 3.22. When comparing the M_y results in Figures 3.22(c) and 3.22(d) to the planar results in Figures 3.11(c) and 3.11(d) it is clear that the moment about the y -axis is not significantly affected by the y displacement. There is less than 6.5% or 0.9 Nmm difference between M_y for the planar and non-planar cases. With regards to M_x and M_z , these moments are the result of the spring ends no longer being colinear. That is, when considering M_z , it follows that a moment should be created about the z -axis when a y displacement at one of the ends of the spring occurs. This is because the shear force (F_x) in the spring now causes a torque about the z -axis. The same follows for the x -axis. With a y displacement the resulting force in the y direction causes a torque about the x -axis. Figure 3.23 shows this equilibrium state. Discrepancies between the numerical and empirical values for the moments can again be attributed to factors such as errors in the representation of the geometry and non-ideal boundary conditions. For example, the discrepancy in magnitude of the moment about the z -axis is possibly due to some torsion in the mounting brackets. That is, when bending the excess wire around the mounting bracket, it is possible that a twist or non-planar torque was placed on the wire in the mounting bracket. This would result in discrepancies between the numerical and empirical results. Also, a trend was noticed with the numerical predictions generally being higher than the empirical measurements. This trend is likely due to the idealized boundary conditions.



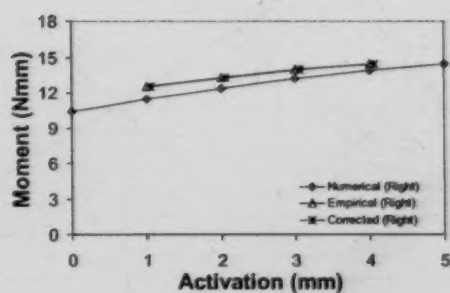
(a) $M_x(left)$



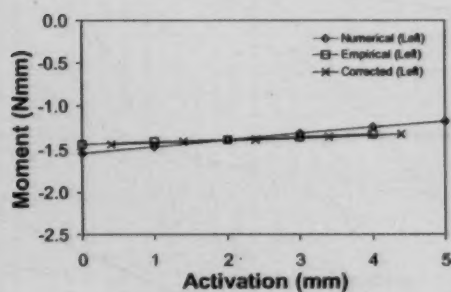
(b) $M_x(right)$



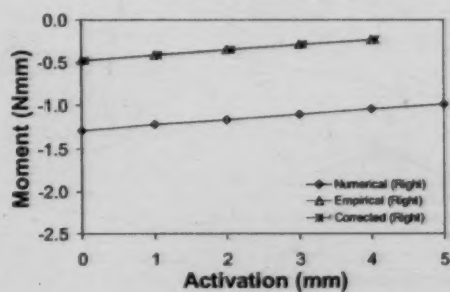
(c) $M_y(left)$



(d) $M_y(right)$

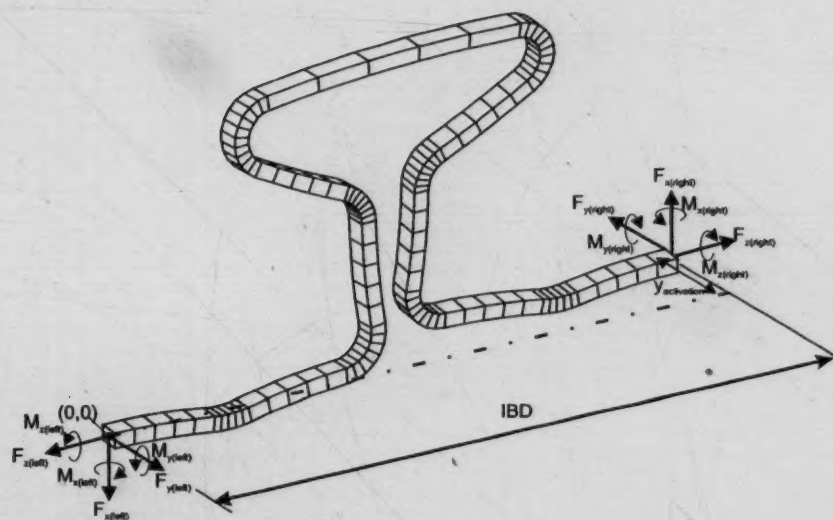


(e) $M_z(left)$



(f) $M_z(right)$

Figure 3.22: T-Spring Moment Results ($y = 2$ mm)



Equilibrium Equations

$$\begin{aligned}
 +\downarrow \Sigma F_x &= F_{x(\text{left})} - F_{x(\text{right})} & +\downarrow \Sigma M_x &= M_{x(\text{left})} - M_{x(\text{right})} - F_{y(\text{right})} \cdot \text{IBD} + F_{z(\text{right})} \cdot y_{\text{activation}} \\
 +\swarrow \Sigma F_y &= F_{y(\text{left})} - F_{y(\text{right})} & +\swarrow \Sigma M_y &= M_{y(\text{left})} - M_{y(\text{right})} + F_{x(\text{right})} \cdot \text{IBD} \\
 +\nearrow \Sigma F_z &= F_{z(\text{left})} - F_{z(\text{right})} & +\nearrow \Sigma M_z &= M_{z(\text{left})} - M_{z(\text{right})} - F_{x(\text{right})} \cdot y_{\text{activation}}
 \end{aligned}$$

Figure 3.23: Equilibrium State

As mentioned in previous sections, of particular concern to the clinician is the moment-to-force ratio created by the orthodontic appliance. Figure 3.24 shows the M/F ratio for T1 with a y displacement of 2 mm (Table 3.12 shows the specific values). Because F_z and M_y were not significantly affected by the out-of-plane displacements, the M/F ratios are relatively unchanged to that of the T1 in a planar scenario. Again, this is clinically advantageous because it means that small misalignments in the mounting brackets do not significantly affect the M/F ratio. Another clinical advantage is that this out-of-plane component does not significantly reduce maximum activation of the spring. The y displacement of 2 mm only resulted in a decrease of ~ 0.4 mm in maximum activation.

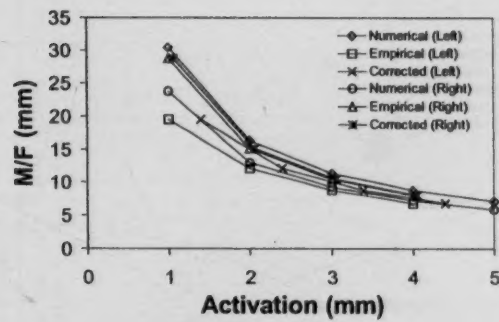


Figure 3.24: T-Spring M/F Ratio ($y = 2$ mm)

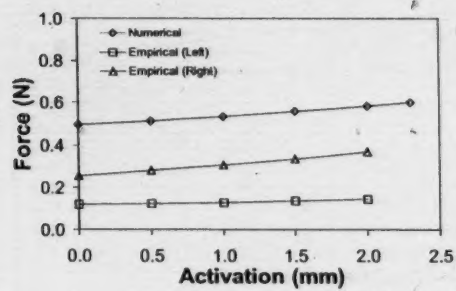
Table 3.12: T-Spring 3D Study M/F Ratios

Activation (mm)	M/F Ratio (mm)					
	Num. L	Num. R	Emp. L	Emp. R	Cor. L	Cor. R
1.0	30.5	23.8	19.4	28.6	-	-
1.1	-	-	-	-	-	28.6
1.4	-	-	-	-	19.4	-
2.0	16.3	13.1	12.0	15.1	-	-
2.1	-	-	-	-	-	15.1
2.4	-	-	-	-	12.0	-
3.0	11.3	9.2	8.7	10.5	-	-
3.1	-	-	-	-	-	10.5
3.4	-	-	-	-	8.7	-
4.0	8.7	7.2	6.8	8.0	-	-
4.1	-	-	-	-	-	8.0
4.6	-	-	-	-	6.8	-
5.0	7.1	5.9	-	-	-	-

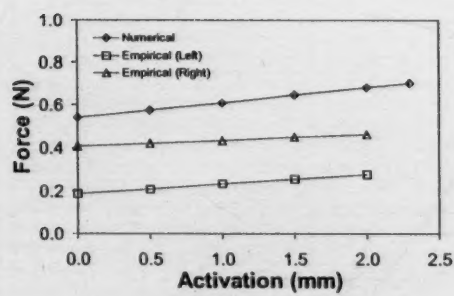
3.5.2 Vertical-Loop Springs

Similar to the T-springs, full three-dimensional tests were completed for the vertical-loop springs (V-springs). No experimental difficulty in finding the neutral position (shown in Figure 2.8) was experienced with the V-springs because no contact between the vertical segments occurred. The results of a full three-dimensional test are shown in Figures 3.25 and 3.26. The test for $x = 1\text{ mm}$ and $y = 1\text{ mm}$ was selected to illustrate the combined effects of these two displacements. The y displacement creates out-of-plane forces and moments (as discussed in the previous section) while the x displacement requires a force in the x direction. This affects F_x and, in turn, M_y . Figure 3.25(c) shows that, similar to the planar results in section 3.4.2, the F_z force is over-predicted numerically. This over-prediction, however, is consistent as F_z is over-predicted by 30% in the non-planar settings and 27% in the planar settings. The rest of Figures 3.25 and 3.26 show that errors are larger than those in the planar situation. As with previous results, the numerical predictions still generally tend to over-predict the actual force and moment levels. Of particular concern clinically are the out-of-plane moments generated by the misalignment. While these were not predicted or measured for the planar cases, significant error is shown for M_x and M_z in the full three-dimensional setting. For example, a significant moment (maximum of $\sim 5\text{ Nmm}$) is predicted about the x -axis when an out-of-plane displacement of $x = 1\text{ mm}$ and $y = 1\text{ mm}$ is used. This is significantly different than the $0.5 - 3.0\text{ Nmm}$ measured experimentally. Such a result is of clinical concern for two reasons. Firstly, if the intention of the orthodontic treatment was to have the mounting brackets aligned, this shows that unwanted misalignments can result in significant out-of-plane moments. Secondly, if the intention was to have this offset to generate out-of-plane moments, these results show that the numerical prediction may not accurately model certain scenarios.

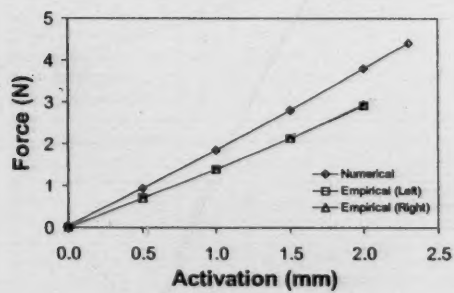
The difference between the modeled M_x , M_y , and M_z and their corresponding numerical predictions can be attributed to several factors. One key factor for this increased error likely is that the two helices present in the geometry create small out-of-plane components. Numerically these helices are simply assumed to be planar but experimentally these helices are not planar and there is contact between the wire at the helices. No contact forces are modeled numerically. The contact also results in friction between the wires which affects the forces as shown. Other factors likely influencing this error are: error in the lengths of sections 1 and 7, the imperfect fit of the wire in the mounting brackets, and the fact that thin beam theory is being used to model the spring when in fact non-thin beam conditions are present. With regards to the fit



(a) F_x

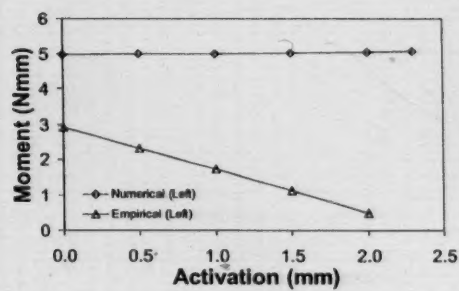


(b) F_y

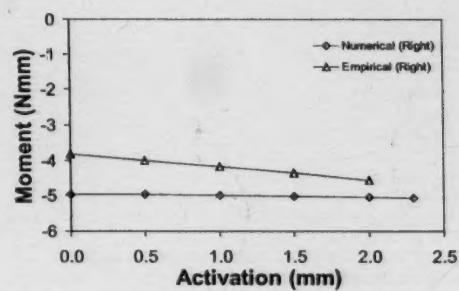


(c) F_z (Tension)

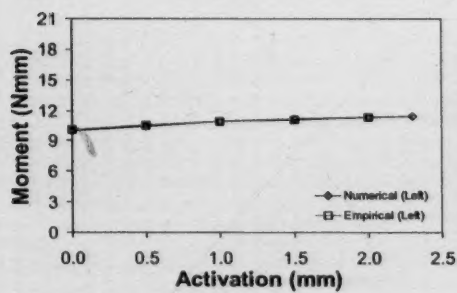
Figure 3.25: Vertical-Loop Spring Force Results ($x = 1\text{mm}$, $y = 1\text{ mm}$)



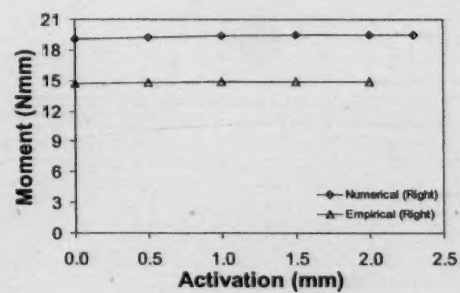
(a) M_x



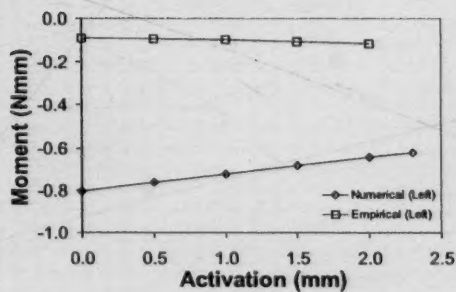
(b) M_x



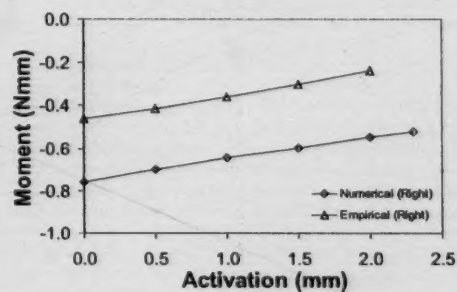
(c) M_y



(d) M_y



(e) M_z



(f) M_z

Figure 3.26: Vertical-Loop Spring Moment Results ($x = 1\text{mm}$, $y = 1\text{mm}$)

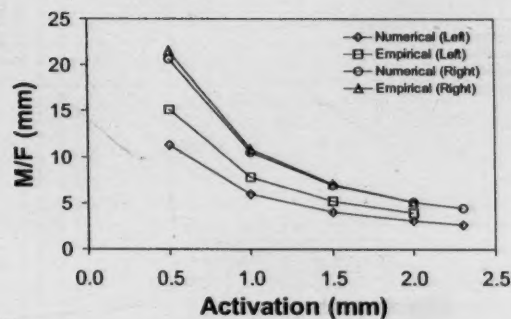


Figure 3.27: Vertical-Loop Spring M/F Ratio ($x = 1\text{mm}, y = 1\text{mm}$)

Table 3.13: V-Spring 3D Study M/F Ratios

Activation (mm)	M/F Ratio (mm)			
	Num. L	Num. R	Emp. L	Emp. R
0.5	11.3	20.7	15.1	21.6
1.0	5.9	10.5	7.8	10.8
1.5	4.0	7.0	5.2	7.0
2.0	3.0	5.1	2.9	5.1
2.5	2.7	4.4	-	-

of the wire in the mounting brackets, not only does this fit lead to torsion or twisting in the mounting brackets, it also results in a non-ideal boundary conditions. This means that the spring arm is not perfectly attached at the bracket and that it can enter the bracket at a slight angle (i.e. the connection is not rigid). Furthermore, the effects of this imperfect fit are likely more prominent in these full 3D cases than in purely planar settings.

Figure 3.27 and Table 3.13 show the M/F ratios of a vertical-loop spring for a complete three-dimensional test. From this figure, it can be seen that the numerical predictions agree quite well with the actual values in this situation. For activations of greater than 1.5 mm (the ~constant M/F ratio region) the discrepancy between numerical and empirical results is less than 1 mm. The numerical method also correctly predicted the presence of different M/F ratios at the left and right ends of the spring. This is clinically important because treatment might call for different M/F ratios at the mesial and distal ends of the spring.

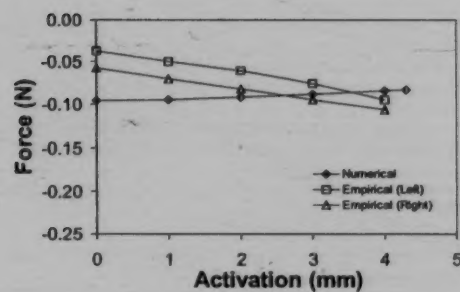
3.5.3 Archial-Loop Springs

Full three-dimensional tests were also completed for the archial-loop springs. The results of a full three-dimensional test ($y = 1$ mm) for A1 are shown in Figures 3.28 and 3.29. When comparing the response of axial force of A1 under planar activations (Figure 3.18(a)) to that shown in Figure 3.28(c), it is clear that the general response of F_z is not affected by out-of-plane displacements. Thirty percent error is consistently found for both the planar and non-planar activations. Furthermore, the slight nonlinearity in the axial force-activation relationship measured experimentally is predicted numerically for both the planar and non-planar scenarios. Of particular interest in Figure 3.28(b) is the "jump" in the *Empirical (Right)* curve between 2.0 and 3.0 mm activation. This is an example of what can occur when the spring slides or twists in the mounting bracket. The "jump" can be seen in Figure 3.29(b) as well.

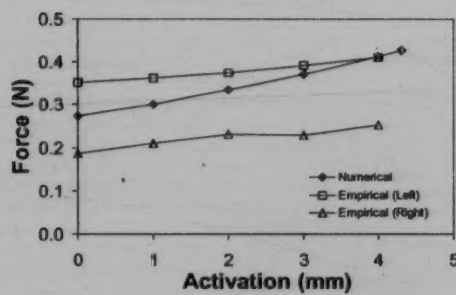
Figure 3.29 shows that there is significant error in the out-of-plane moments. One reason for this increased error is that, similar to the vertical-loop spring, the two helices present in the geometry create small out-of-plane components. This is because numerically these helices are simply assumed to be planar but experimentally these helices are not planar. Also, there is contact between the wire at the helices. This results in contact forces and friction between the wires.

Unlike the vertical-loop three-dimensional results, the magnitude is larger for some empirical measurements than the numerical modeling. For example, the M_x *Empirical Left* results are approximately double those of the numerical predictions. The M_x *Empirical Right* results are even of different sign. To further confirm the measured values for $M_{x(Right)}$, an equilibrium check was preformed. Using the results from both the left and right tests, moments were summed about x -axis for the spring. The result is shown in Figure 3.30. While it is believed that the helices are a major factor of this error, it is also believed the placement of the spring in the mounting brackets influenced the results as it did the planar T-spring investigated in Section 3.4.1.

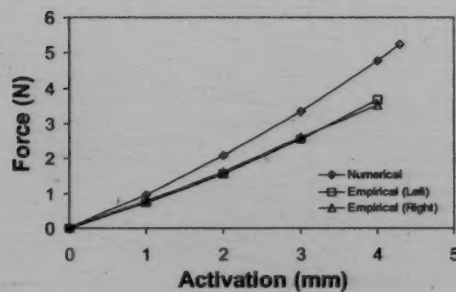
The M/F ratio for this archial-loop spring with an out-of-plane activation is shown in Figure 3.31. Specific values of the M/F ratios are listed in Table 3.14. As expected the M/F ratio is lower numerically. This is because the force is numerically over-predicted. However, at an activation of 4 mm the numerical modeling is still able to predict the M/F ratio within ~ 1 mm.



(a) F_x

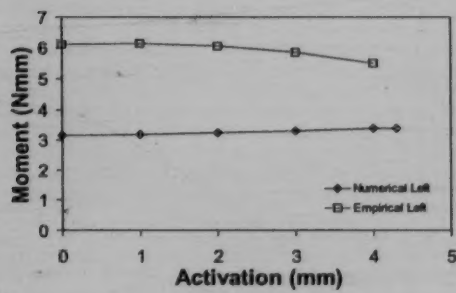


(b) F_y

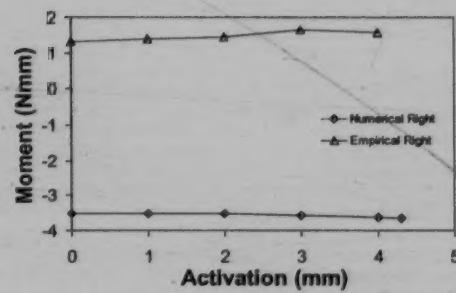


(c) F_z (Tension)

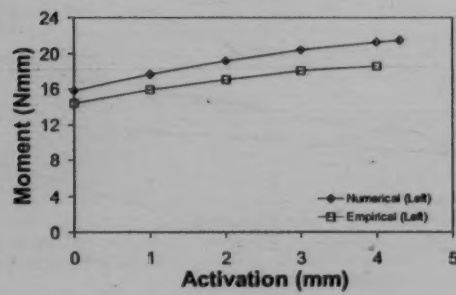
Figure 3.28: Archial-Loop Spring Force Results ($y = 1$ mm)



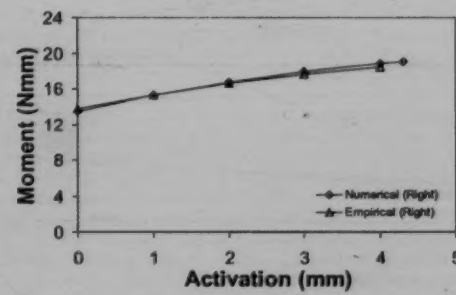
(a) M_x



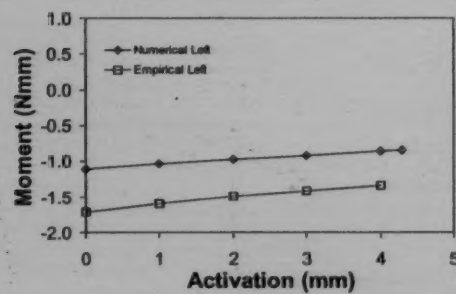
(b) M_x



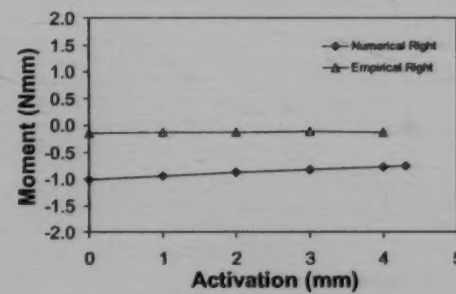
(c) M_y



(d) M_y



(e) M_z



(f) M_z

Figure 3.29: Archial-Loop Spring Moment Results ($y = 1$ mm)

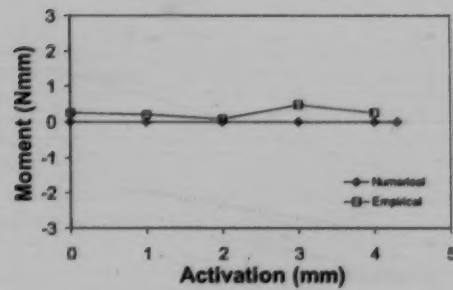


Figure 3.30: Archial-Loop Spring Equilibrium Check about x -Axis

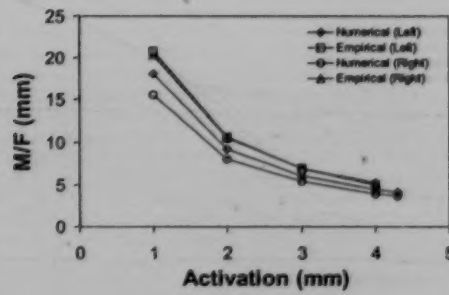


Figure 3.31: Archial-Loop Spring Moment-to-Force Ratio ($y = 1$ mm)

Table 3.14: A-Spring 3D Study M/F Ratios

Activation (mm)	M/F Ratio (mm)			
	Num. L	Num. R	Emp. L	Emp. R
1.0	18.1	15.6	20.8	20.3
2.0	9.2	8.1	10.1	10.5
3.0	6.1	5.4	6.9	6.9
4.0	4.4	4.0	5.0	5.2
4.3	4.1	3.7	-	-

3.6 Force Systems from Non-Planar Springs

As discussed in Chapter 1, another important aspect of clinical orthodontics is consideration of the moment-to-force ratio in the occlusal plane (See Figure 1.6). One means of preventing unwanted rotation in the occlusal plane is to design the orthodontic appliance so that it is capable of producing a moment in the occlusal plane which counteracts this rotation. This involves a non-planar configuration of the spring's initial geometry as shown in Figure 3.32. This anti-rotation T-spring is similar to the planar T-springs modeled in previous sections of this thesis. However, unlike the other T-springs, there are out of plane components to the original geometry. Figure 3.32 shows that sections 5 and 13 have some initial twist which results in the arms of the spring having out-of-plane components. The digitized geometry used to numerically model this spring is given in Table 3.15; however, with no accurate method of measuring the angle of twist in sections 5 and 13, it was difficult to accurately digitize the spring geometry. A digital scanner was still used to measure the in-plane geometry of the spring but no scan was possible to measure the out-of-plane component. To determine the out-of-plane angle, a caliper was used to measure the out-of-plane height of the spring. This distance was then used to calculate the required angle of twist.

Figure 3.33 shows that, even with the out-of-plane components, the numerically determined spring constant of 0.38 N/mm agrees within 5% of the empirically measured spring constant of 0.40 N/mm. These results are for the scenario in which the anti-rotation T-spring was placed in a planar situation and activated along the z-axis. Furthermore, the results show that there is less than 0.04 N difference between the numerical and empirical values for F_y and 0.2 N difference for F_x . It should also be noted that, similar to the initially planar T-springs investigated in Section 3.4.1 and 3.5.1, the neutral position could not be found directly. This is because contact occurred between sections 6 and 12 before the true neutral position could be reached. This is the reason for the corrected curves shown.

Because it was difficult to determine the angle of twist in sections 5 and 13, a parametric study was conducted to investigate the effect of this parameter. The results of this variation are shown in Figure 3.34. The angle was varied from 10 to 15 degrees. An important fact indicated from these results is that both F_x and F_z do not change for small changes in twist angle is clinically beneficial.

The results of the moment measurements for this test involving the non-planar spring are shown in Figure 3.35. Figure 3.35(a) shows that there is

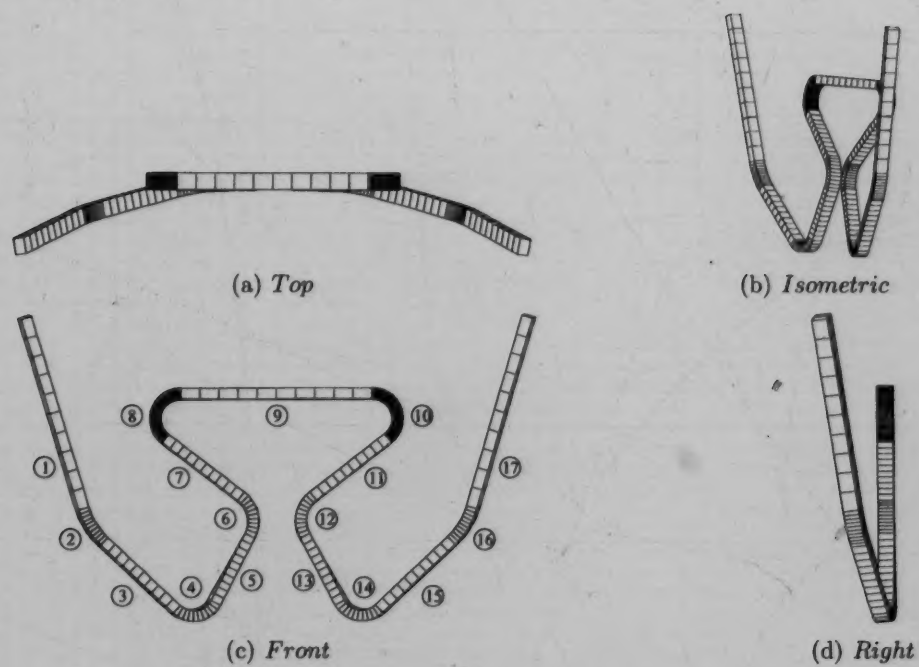
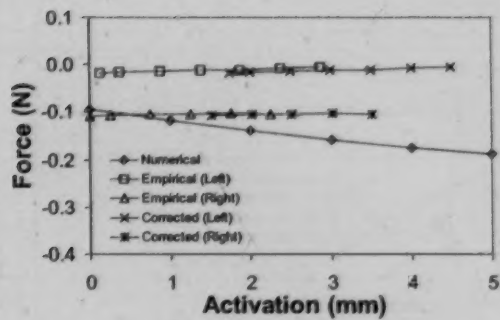


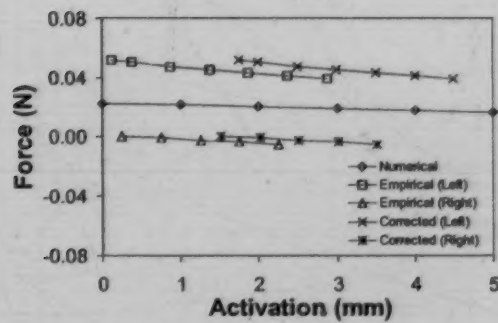
Figure 3.32: Anti-Rotation T-Spring Geometry

Table 3.15: Anti-Rotation T-Loop Spring Geometry

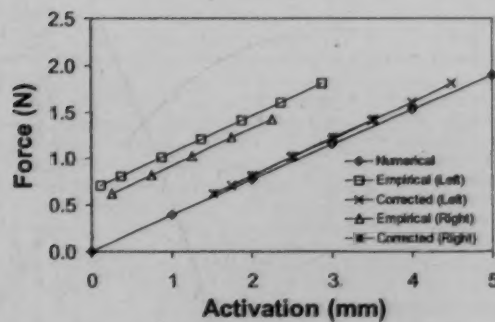
Section	Straight, Curved, or Twisted	Length (mm)	Radius (mm)	Angle (°)
1	Straight	4.00	-	-
2	Curved	-	1.82	29.5
3	Straight	4.14	-	-
4	Curved	-	0.89	109.8
5	Twisted	3.15	-	12.0
6	Curved	-	1.24	73.3
7	Straight	3.40	-	-
8	Curved	-	1.00	142.4
9	Straight	7.73	-	-
10	Curved	-	0.97	140.7
11	Straight	3.66	-	-
12	Curved	-	1.63	73.0
13	Twisted	3.16	-	12.0
14	Curved	-	1.00	109.6
15	Straight	3.94	-	-
16	Curved	-	1.30	28.4
17	Straight	6.00	-	-



(a) F_x

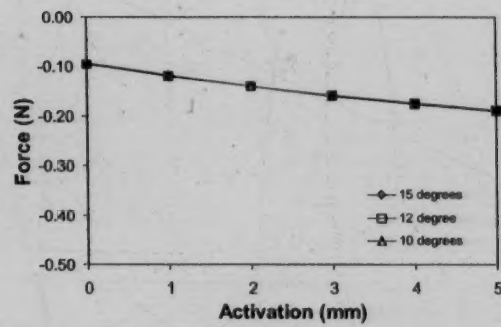


(b) F_y

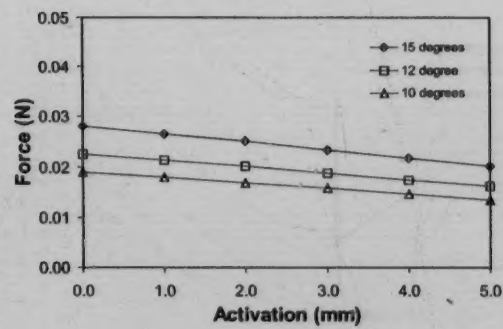


(c) F_z (Tension)

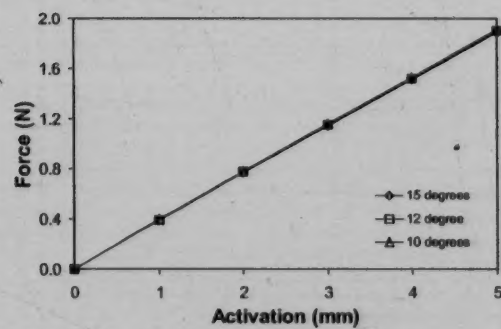
Figure 3.33: Anti-Rotation T-Spring Force Data



(a) F_x



(b) F_y



(c) F_z (Tension)

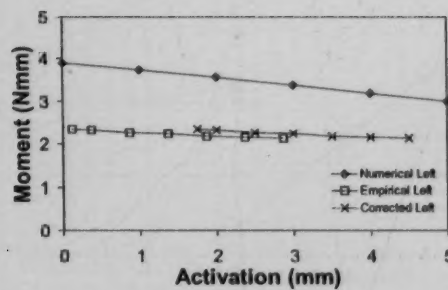
Figure 3.34: Anti-Rotation T-Spring Force Data for Varying Twist Angles

significant error (~ 1.6 Nmm or $\sim 70\%$) in the predictions of M_x . One likely reason for this is the error in the geometry digitization. Figure 3.35, however, does show that the out-of-plane geometry does not affect the results of M_y . The ~ 2 Nmm (12%) error in M_y is consistent with that of both the planar and three-dimensional tests. This error can likely be attributed to errors in the lengths of sections 1 and 17.

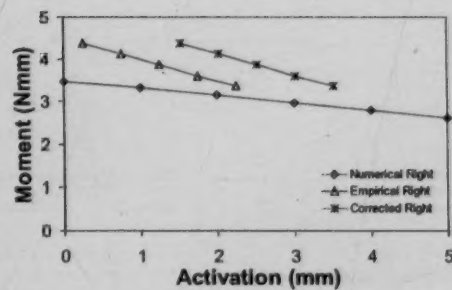
Similar to the force results, a study was conducted to determine the effect of the twist angles on the moment predictions. The results of this are shown in Figure 3.36. From this it is clear that small changes in twist angle affect M_x and M_z . For example, the moment about the x -axis changed from 4.9 Nmm to 3.3 Nmm at zero activation (at the left end of the spring) for a change in twist angle from 15° to 10° . With regards to M_y , it is not affected by small changes in twist angle. This is shown in Figure 3.36(b).

Error in digitization is not the only cause of this inaccurate modeling. As discussed in previous sections, the non-ideal boundary conditions also likely contribute to the modeling error. It should also be noted that another possible source of error in the geometry digitization is that a small amount of twist might have been present in sections 4 and 14 as well as section 5 and 13. No numerical investigation regarding this was conducted however.

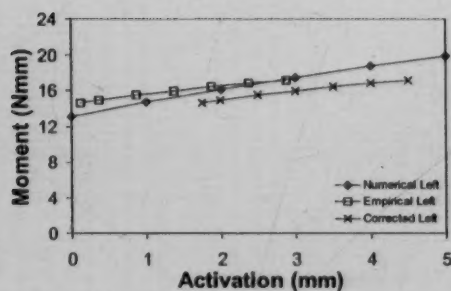
Figure 3.37 shows the M/F ratios in both the occlusal and sagittal planes for the anti-rotation T-spring tested. Tables 3.16 and 3.17 list the specific M/F ratio data. As expected, there is significant error in the M_x/F ratio for the left end of the spring because of the error in M_x . For example, the *Corrected Left* plots show that for an activation of 3.5 mm the anti-rotation spring generates a sagittal M/F ratio of 9.4 mm and an occlusal M/F ratio of 1.2 mm. These are differences of 1.6 mm and 0.7 mm from the numerical values respectively. This represents a 13% and 58% error respectively. Moreover, if the modeling were correct, it would lead to the fact that this spring, used under these conditions, would not be desirable for orthodontic treatment requiring translation. This is because a sagittal M/F ratio of ~ 8.5 mm and occlusal M/F ratio of ~ 3.5 mm is desired for translation. Figure 3.37 also indicates that there is no activation in which both of these are met; however, the numerical investigation reveals that the M/F ratios in both the sagittal and occlusal planes do approach translation.



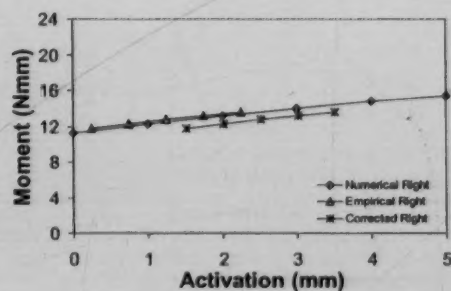
(a) M_x



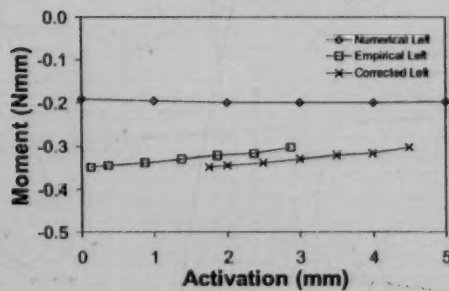
(b) M_x



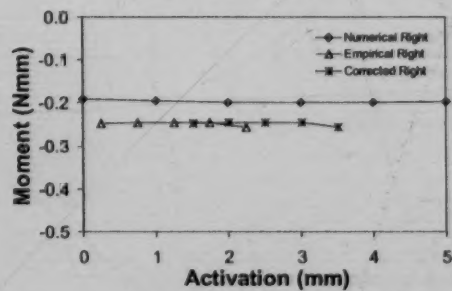
(c) M_y



(d) M_y

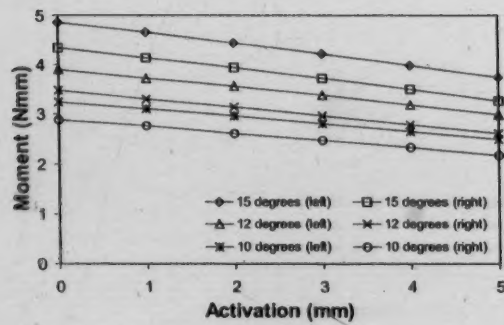


(e) M_z

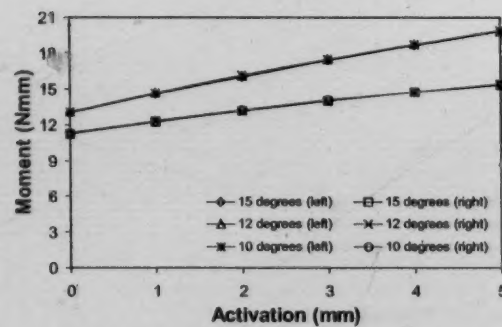


(f) M_z

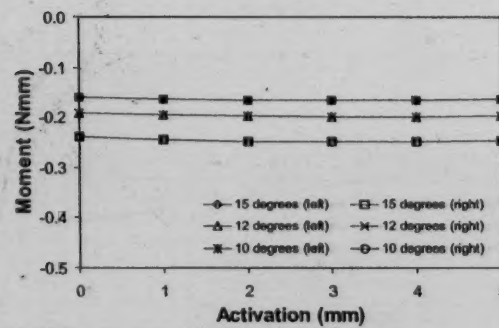
Figure 3.35: Anti-Rotation T-Spring Moment Data



(a) M_x

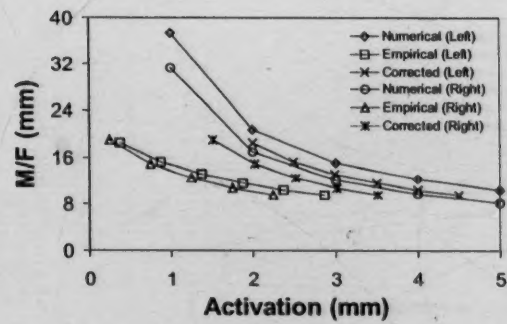


(b) M_y

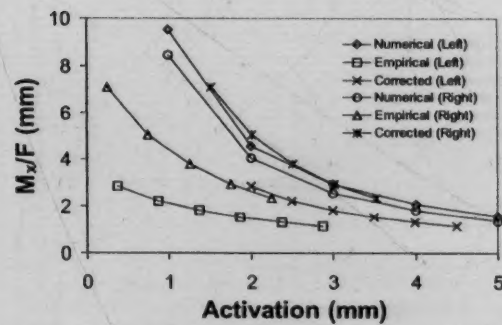


(c) M_z

Figure 3.36: Anti-Rotation T-Spring Moment Data for Varying Twist Angles



(a) M/F - Sagittal Plane



(b) M/F - Occlusal Plane

Figure 3.37: Anti-Rotation T-Spring M/F Ratios

Table 3.16: Anti-Rotation T-Spring M/F Ratio: Sagittal Plane

M/F Ratio (mm) Activation (mm)					
Num. L	Num. R	Emp. L	Emp. R	Cor. L	Cor. R
9.5	8.5	2.9	7.1	2.9	7.1
1.0	1.0	0.4	0.3	2.0	1.5
4.6	4.1	2.2	5.0	2.2	5.0
2.0	2.0	0.9	0.8	2.5	2.0
2.9	2.6	1.8	3.8	1.8	3.8
3.0	3.0	1.4	1.3	3.0	2.5
2.1	1.8	1.5	2.9	1.5	2.9
4.0	4.0	1.9	1.8	3.5	3.0
1.6	1.4	1.3	2.4	1.3	2.4
5.0	5.0	2.4	2.3	4.0	3.5
-	-	1.2	-	1.2	-
5.5	5.5	2.9	2.8	4.5	4.0

Table 3.17: Anti-Rotation T-Spring M/F Ratio: Occlusal Plane

M/F Ratio (mm) Activation (mm)					
Num. L	Num. R	Emp. L	Emp. R	Cor. L	Cor. R
37.4	31.4	18.5	19.1	18.5	19.1
1.0	1.0	0.4	0.3	2.0	1.5
20.8	17.0	15.3	15.0	15.3	15.0
2.0	2.0	0.9	0.8	2.5	2.0
15.1	12.2	13.2	12.5	13.2	12.5
3.0	3.0	1.4	1.3	3.0	2.5
12.3	9.7	11.6	10.7	11.6	10.7
4.0	4.0	1.9	1.8	3.5	3.0
10.5	8.1	10.4	9.5	10.4	9.5
5.0	5.0	2.4	2.3	4.0	3.5
-	-	9.5	-	9.5	-
5.5	5.5	2.9	2.8	4.5	4.0

Chapter 4

A Comparative Study: Archial-Loop Springs vs. T-Loop and Vertical-Loop Springs

In this chapter the mechanical behavior of the archial-loop orthodontic spring design is investigated more thoroughly. The archial-loop spring, a modification of the traditional vertical-loop spring design, involves curved end sections as opposed the straight sections in the vertical-loop design. Such a design is a relatively new concept of which little analysis has been documented. As a means of comparison, the mechanical behavior of the archial design is compared to the more commonly used T- and vertical-loop spring designs. Planar scenarios are investigated in this comparison, as this is most applicable to clinical orthodontics.

For this investigation, properties such as force, moment, moment-to-force (M/F) ratio, and activation range are compared. Force is an important property because sufficient force must be applied to the tooth to cause translation and/or rotation. Similar to force, moment is an important property because the moment generated by the spring can be used to cause tooth rotation or to counteract the rotation caused by the force. Together, moment and force combine to form a resultant M/F ratio. This defines the type of tooth movement that the spring will cause. Activation range is also an important property because it represents the spring lengths at which the spring can be used to cause the desired tooth movement.

Only numerical results are considered in this study. While empirical results would further substantiate the conclusions drawn from the numerical results,

the results of Chapter 3 show that the numerical procedure can consistently predict the general mechanical behavior of the springs in planar scenarios. Three-dimensional effects are not discussed.

4.1 Spring Geometries

The spring geometries used in this comparison are shown in Tables 4.1, 4.2, and 4.3. Symmetric geometries are used as means of simplifying the geometry and force systems. Figure 4.1 shows the relative sizes and shapes of these three designs. These specific geometries were selected because each reaches its maximum activation at an inter-bracket distance (IBD) of approximately 22.0 mm.

Table 4.1: Archial-Loop Spring Geometry for Comparison Study

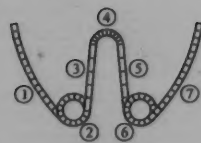
Section	Straight or Curved	Length (mm)	Radius (mm)	Angle (°)
1	Curved	-	13.00	36
2	Curved	-	1.00	490
3	Straight	5.00	-	-
4	Curved	-	1.00	170
5	Straight	5.00	-	-
6	Curved	-	1.00	490
7	Curved	-	13.00	36

Table 4.2: Vertical-Loop Spring Geometry for Comparison Study

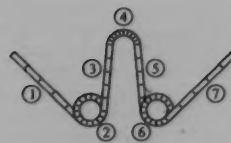
Section	Straight or Curved	Length (mm)	Radius (mm)	Angle (°)
1	Curved	7.00	-	-
2	Curved	-	1.00	490
3	Straight	5.00	-	-
4	Curved	-	1.00	170
5	Straight	5.00	-	-
6	Curved	-	1.00	490
7	Curved	7.00	-	-

Table 4.3: T-Loop Spring Geometry for Comparison Study

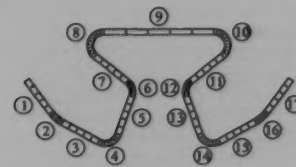
Section	Straight or Curved	Length (mm)	Radius (mm)	Angle (°)
1	Straight	3.00	-	-
2	Curved	-	2.50	25
3	Straight	3.00	-	-
4	Curved	-	1.00	100
5	Straight	3.00	-	-
6	Curved	-	1.00	70
7	Straight	3.00	-	-
8	Curved	-	1.00	141
9	Straight	8.00	-	-
10	Curved	-	1.00	141
11	Straight	3.00	-	-
12	Curved	-	1.00	70
13	Straight	3.00	-	-
14	Curved	-	1.00	100
15	Straight	3.00	-	-
16	Curved	-	2.50	25
17	Straight	3.00	-	-



(a) Archial-Loop Spring



(b) Vertical-Loop Spring



(c) T-Spring

Figure 4.1: Comparison Study: Orthodontic Spring Geometries
(Scale $\times 2$)

4.2 Activation Range

Due to the variation in sizes of spaces to be closed between teeth, activation range is an important property of segmented appliances. This is because a larger activation range results in sufficient moments and forces being applied to the tooth over a larger translation distance. Consequently, the orthodontist is required to perform fewer adjustments during treatment. That is, as tooth movement (usually translation) progresses, the IBD decreases. Therefore, the activation of the spring decreases. Less force is then applied to the tooth. Lower activation also results in a higher M/F ratio generated by the spring. In fact, if the activation becomes small enough the M/F ratio increases and causes root movement. To compensate for this, the orthodontist shortens the lengths of sections 1 and 7 (or 17) by drawing the wire further through the mounting bracket. This increases the activation of the spring.

To determine activation range two criteria are considered. The first criteria is the amount of axial activation from its neutral position that the spring can sustain prior to yielding. This distance does not directly give the activation range however. As discussed in Chapter 1, a desired M/F ratio is needed to produce the desired type of tooth movement. Therefore, activation range is further restricted to the range in which the orthodontic spring is capable of generating appropriate M/F ratios. This means that a flatter M/F ratio curve is more desirable in most situations. Figure 4.2 shows the M/F ratios of all three orthodontic spring designs for various inter-bracket distances (IBDs). Table 4.4 in Section 4.4 lists the M/F ratio data. Similar to the spring repeatability study in Section 3.3, IBD is used instead of activation because it is more clinically applicable. This is because it is clinically more important to know the force, moment, and M/F ratio the spring generates for a given IBD than for a specific activation. From these results it is clear that, by curving sections 1 and 7 of the archial-loop spring, no gain in activation range is achieved when compared to the vertical-loop spring. However, the curved end sections do increase the M/F ratio so that this ratio is closer to 8.5 mm than the vertical-loop spring throughout its activation.

4.3 Force, Moment, Moment-to-Force Ratio

Figure 4.3 compares the axial force versus inter-bracket distance of the A-spring to the V- and T-springs. As expected, the vertical-loop spring is stiffer and delivers higher forces than the T-spring. Similarly, the archial-loop spring is stiffer than the T-spring. This is not surprising given that it is a modifica-

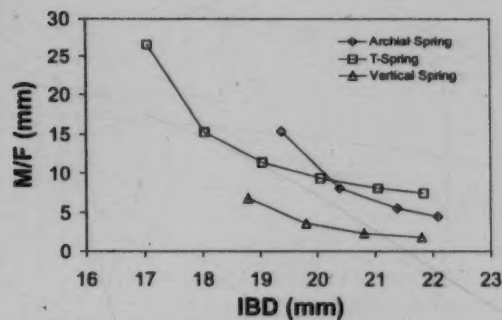


Figure 4.2: M/F Ratio Comparison

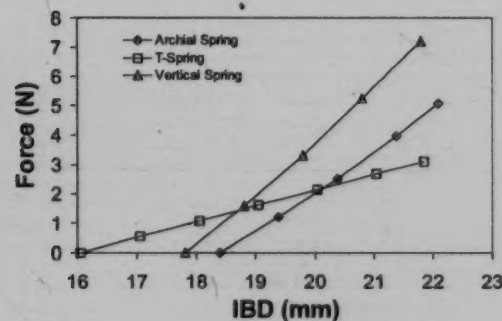


Figure 4.3: Force Comparison

tion of the V-spring. By curving sections 1 and 7 of the archial-loop spring, however, the force at each IBD is significantly reduced. At the maximum activations (IBD of ~ 22 mm) the archial-loop spring's axial force is approximately 2 N less than the vertical-loop spring's axial force. It should be noted that, while both the archial-loop and vertical-loop spring force vs. IBD relationships are shown to be significantly steeper than the T-spring force vs. IBD relationship, results from Chapter 3 show that the numerical modeling overpredicts the forces for the archial and vertical-loop springs by approximately 27 – 30%. This is likely because of the presence of the helices.

Allowing sections 1 and 7 to have some initial curvature has a significant effect on the in-plane moment. This is shown in Figure 4.4. The archial-loop spring's moment about the y -axis is increased compared to the vertical-loop

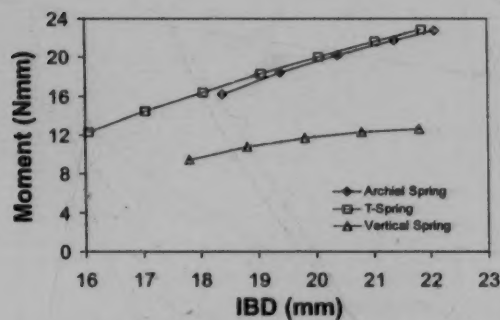


Figure 4.4: Moment (M_y) Comparison

spring. Rather than generating moments of between 9.5 and 12.5 Nmm, similar to the vertical-loop spring, the archial-loop spring generates moments between 16.2 and 22.7 Nmm. This is similar to that of the T-spring. Such a result is a direct result of the initial curvature to section 1 and 7 for the archial-loop spring.

The effects, with regards to M/F ratio, of adding an initial curvature to sections 1 and 7 are shown in Figure 4.2. As expected, since the presence of the initial curvature to the end sections affects both the force and moment generated, it follows that the M/F ratio of the archial-loop spring is different than that of the vertical-loop spring. Figure 4.2 shows that, for the spring designs considered, the A-spring's M/F ratio is more than double the M/F ratio of the V-spring for the same IBD. The archial-loop spring's M/F ratio is also closer to the desired value of 8.5 mm for a wider activation range. When compared to the T-spring, it is clear that the M/F ratio of the T-spring is still more desirable. This is because the M/F ratio curve for the T-spring is more constant for a wider activation and still close to 8.5 mm. Still, the addition of an initial curvature to the end sections in the archial-loop spring clearly results in significant improvement over the basic vertical-loop spring.

4.4 Geometry Modification

To further improve the mechanical behavior of the A-spring, the addition of helical section at section 4 is considered in this section. It should be noted that no empirical testing was conducted with such a geometry. To model this geometry, the A-spring geometry presented in Table 4.1 is used with a

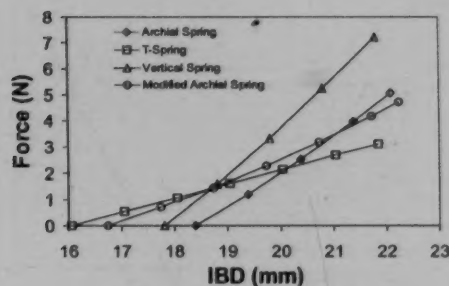


Figure 4.5: Force Comparison with Geometry Modification

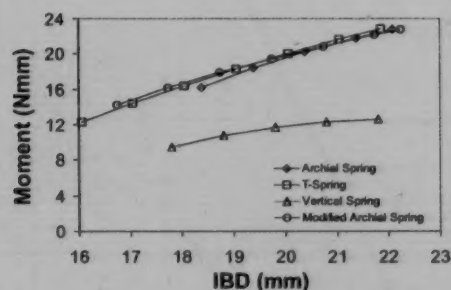


Figure 4.6: Moment (M_y) Comparison with Geometry Modification

modification to section 4. To create the apical helix, 360° is added to the angle of section 4. The corresponding effects on spring force and moment of this helix are shown in Figures 4.5 and 4.6 respectively. Figure 4.5 shows that the presence of the apical helix softens the springs. This agrees with the results of adding a apical helix to basic vertical-loop spring discussed in Faulkner, MG et al. (1991). Figure 4.6 shows that the apical helix does not result in a significant affect on the moment generated however.

This leads to the change in M/F ratio as shown in Figure 4.7. From the results shown in Figure 4.7 and Table 4.4, it is clear that the presence of the apical helix results in a flatter or more constant M/F ratio. This increases the useable activation range because the M/F ratio remains closer to 8.5 mm for a wider range. It also allows for more predictable and controllable tooth movement.

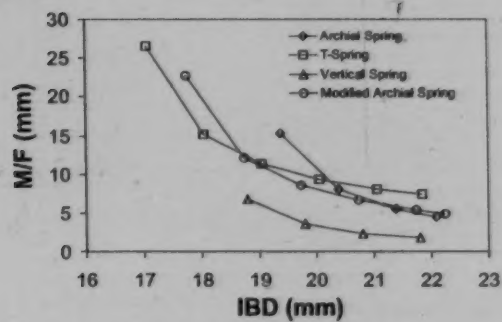


Figure 4.7: M/F Ratio Comparison with Geometry Modification

Table 4.4: M/F Ratio Comparison

M/F (mm) IBD (mm)			
V-Spring	T-Spring	A-Spring	Mod. A-Spring
-	26.5	-	22.7
17.0	17.0	17.0	17.7
-	15.2	-	12.2
18.0	18.0	18.0	18.7
6.8	11.4	15.4	8.5
18.8	19.0	19.4	19.7
3.5	9.3	8.1	6.6
19.8	20.0	20.4	20.7
2.3	8.1	5.5	5.3
20.8	21.0	21.4	21.7
1.7	7.4	4.5	4.8
21.8	22.0	22.4	22.2

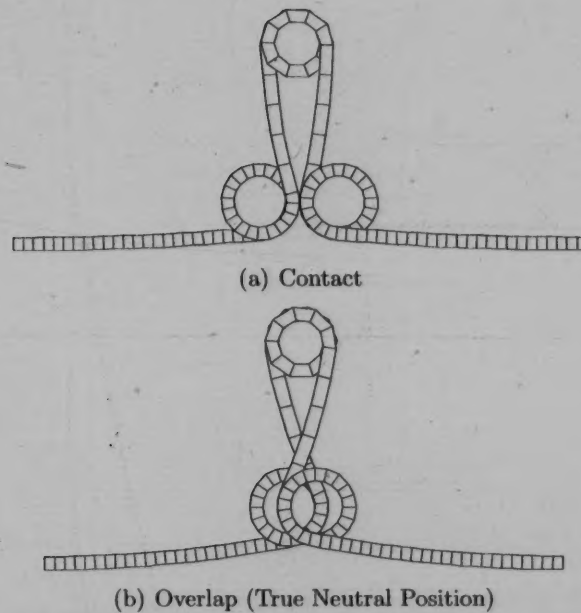


Figure 4.8: Modified Archial-Spring Neutral Position / Contact

Figures 4.5-4.7 also show that the overall activation range of the modified archial-loop spring is greater than that of the archial-loop spring. Numerically, the modified spring was found to have an activation range from a neutral position of 16.7 mm to just over 22 mm, whereas the archial-loop spring's neutral position was 18.4 mm. It should be noted, however, that while the numerical results appear to increase the overall activation range, this likely is not the case empirically. This is because the numerical results indicate that contact between sections 2 and 6 would likely occur before the true neutral position is reached. Figure 4.8 demonstrates this. This is similar to the problem with the T-springs discussed in Section 3.4.1. Numerically, this overlap distance was found to be approximately 1.6 mm, and thus, the gained activation range would be ~ 1.6 mm shorter empirically than the predicted numerically.

4.5 Clinical Implications

The initial curvature of sections 1 and 7 in the design of the archial-loop spring makes the design an appropriate replacement to the T-spring. This is not only

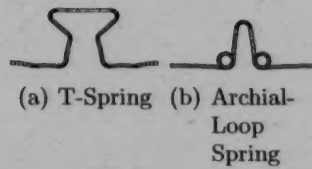


Figure 4.9: Archial-Loop Spring vs. T-Spring Size Comparison
(To Scale)

because the resultant mechanical properties are comparable to the T-spring, but because there are aspects of the A-spring that sometimes make it more desirable than the T-spring. One of these advantages is the fact that it is a smaller spring. Figure 4.9 shows this as the apical portion of the spring has a smaller area than that of the T-spring. This can make it more comfortable for the patient because there is less wire in the vestibule of the cheek. Furthermore, the spring may be easier for the orthodontist to place in the mouth.

Chapter 5

Conclusions

5.1 Summary

Crowding of the orthodontic arch, a common occurrence, is often treated by extracting posterior teeth (such as bicuspid). Extraction sites are created in strategic areas of the dental arch to permit the relocation of the crowded teeth. To accomplish this displacement, space closure is necessary.

The technique of space closure involves the clinical application of several biomechanical concepts. For example, the generation and application of forces and moments are key to successful treatment. Therefore, it is important for the clinician to understand these concepts. Such comprehension helps the clinician minimize and/or eliminate unknown variables of the orthodontic treatment. For this reason, understanding the forces and moments generated by the orthodontic appliances necessary for space closure was the focus of this work.

In this work the numerical modeling of the three-dimensional force systems generated by orthodontic retraction appliances was evaluated by comparing numerical and empirical results. For this investigation, a previously developed numerical procedure (Raboud, D et al., 1996) was used for all numerical modeling. Experimental results were obtained using an apparatus that controlled the deformation of the spring and measured the resulting force systems. Additionally, an evaluation of a modified vertical-loop retraction spring (known as the archial-loop spring) and an evaluation of the variability in mechanical properties of clinically identical springs were conducted.

To numerically model the orthodontic springs, the geometry and material properties of the springs were required. Spring geometry was determined using a digital scanner and measurement. Young's modulus of the titanium molybdenum alloy used to make the orthodontic springs was measured and found to

be 62.5 ± 0.7 GPa from tensile testing. All other properties were determined from material specifications.

To experimentally determine the three-dimensional force systems from the orthodontic springs, 3 different spring designs (T-loop spring, vertical-loop spring, and archial-loop spring) were used in planar and non-planar scenarios. The non-planar results were used in the full three-dimensional analysis of the orthodontic appliances. This analysis showed both the clinical implications of out-of-plane activations of the springs as well as the accuracy of the numerical modeling in fully three-dimensional settings. The planar analysis was used to compare clinically applicable properties between each of the spring designs and springs of the same design.

The spring repeatability study included in Chapter 3 of this work, investigates the forces and moments generated from clinical identical retraction springs. Five T-loop springs and 5 archial-loop springs were used in this study which only involved planar situations.

Chapter 4 of this work investigates more thoroughly the archial-loop spring design. Spring properties such as moment-to-force ratio and activation range were considered. These properties were then compared to the traditional T-loop and vertical-loop spring designs.

5.2 Conclusions

- The planar components of orthodontic spring geometry can be measured using a digital scanner; however, non-constant radii were approximated.
- Variability does exist between clinically identical springs. Error in spring placement and differences in geometry can lead to variability in force, moment, and moment-to-force ratio. For example, for the 5 T-springs investigated the empirical M/F ratio varied from 7.7 mm to 10.4 mm at an IBD of 25 mm. The variability in the 5 archial-loop springs investigated resulted in a maximum difference of 1.5 N, 1.3 Nmm, and 2 mm for the axial force, moment, and M/F ratio respectively at an IBD of 25 mm.
- A previously developed numerical procedure (Raboud, D et al., 1996) can be used to model the mechanical behavior of orthodontic retraction springs; however, numerical predictions are less reliable for the out-of-plane moments in the full three-dimensional setting compared to planar results measured in planar settings. For example, the error for M_x was as much as 1.7 (68%), 4.5 (950%), and 2.9 Nmm (316%) for the T-,

vertical-, and archial-loop springs respectively in full three-dimensional scenarios whereas the error in M_y in a planar setting was found to be consistently over-predicted by $< 20\%$.

- The errors for the vertical-loop and archial-loop springs are larger than for the T-loop springs due to presence of the helices. These helices result in non-predicted out-of-plane components. This is likely due to the contact and friction forces present and the initial out-of-plane geometry of the helices themselves.
- Errors in the numerical predictions result from not only the error in geometry digitization but also from the non-ideal boundary conditions and error in the measured lengths of spring arms. For example, for one of the T-springs investigated the prediction of the applied moments at the left and right ends of spring changed from 16.8 Nmm to 15.2 Nmm at the left end and 13.8 Nmm to 13.4 Nmm at the right for an end length adjustment of 1.0 mm and 0.7 mm respectively. This significantly improved agreement with the empirical left and right values of 14.7 Nmm to 13.5 Nmm respectively without significantly affecting the axial force prediction.
- In a full three-dimension scenario the numerical model is consistently able to predict the axial force of the T-spring within 4%, vertical-loop spring within 30%, and archial-loop spring within 30%. One reason for this difference is the presence of the helices.
- For springs of similar design, the archial-loop spring applies a larger moment about the y -axis (between 16.2 and 22.7 Nmm) than a corresponding vertical-loop spring (between 9.5 and 12.5 Nmm). This, in turn, increases the moment-to-force ratio applied to tooth and results in a more desirable retraction spring when compared to the vertical-loop spring.
- The M/F ratio of the T-spring is still more desirable than that of the archial-loop spring because the M/F ratio is more constant. However, the archial-loop spring is a smaller spring which has clinical advantages.
- The inclusion of an apical helix softens the archial-spring but it does not significantly increase the activation range of the spring. Such a helix decreased the slope of the force-activation curve from 1.4 N/mm to 0.86 N/mm. This makes the archial-loop spring more comparable to the T-spring (0.53 N/mm).

5.3 Future Work

While this study is felt to be a very good starting point for determining the numerical accuracy of the three-dimensional force systems from orthodontic retraction springs, it is believed that future studies are still required. Specifically, one area for more in-depth work is with the use of three-dimensional springs. This study primarily focused on using initially planar springs in three-dimensional settings. However, as discussed in Section 3.6, it is possible to have a spring with an initial three-dimensional geometry used in either a planar or three-dimensional setting. Such a spring is of clinical interest because the out-of-plane geometries can be designed to produce specific M/F ratios in the occlusal plane. Future three-dimensional analysis could also include investigating offset angles in the mounting brackets instead of the offset distances considered in this work. Investigations could also involve a parametric study to examine different radii of the curved end sections in the archial-loop design.

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Appendix A

Results

This appendix includes the results of both two- and three-dimensional tests for the springs investigated as part of this work. No new conclusions are drawn from these results rather they are presented to further substantiate results by showing the different scenarios the orthodontic springs were tested and modeled in. Results from T-springs, anti-rotation T-springs, vertical-loop springs, and archial-loop springs are included.

A.1 T-Springs

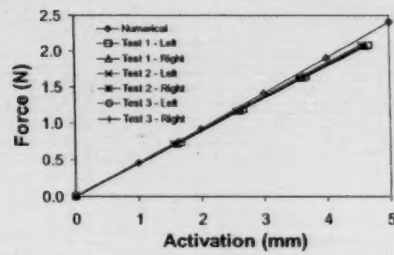
Results from the T-spring tests are shown in this section. When necessary, the empirical results are only plotted with the activation shift already included. Tables A.1 and A.2 show the T-spring geometries. It should also be noted that the left and right ends were not tested for every spring.

Table A.1: T-Spring Geometries

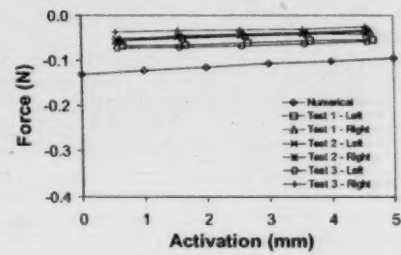
Section	Straight or Curved	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Straight	T1	6.00	-	-
		T2	4.50	-	-
2	Curved	T1	-	2.13	30.1
		T2	-	2.12	30.2
3	Straight	T1	4.26	-	-
		T2	4.12	-	-
4	Curved	T1	-	1.15	98.9
		T2	-	1.04	104.9
5	Straight	T1	2.97	-	-
		T2	2.84	-	-
6	Curved	T1	-	1.06	88.6
		T2	-	1.11	80.5
7	Straight	T1	2.94	-	-
		T2	2.70	-	-
8	Curved	T1	-	1.11	145.3
		T2	-	1.00	143.9
9	Straight	T1	7.43	-	-
		T2	8.23	-	-
10	Curved	T1	-	0.98	143.2
		T2	-	0.80	139.8
11	Straight	T1	3.21	-	-
		T2	3.69	-	-
12	Curved	T1	-	1.03	85.2
		T2	-	1.20	79.4
13	Straight	T1	3.06	-	-
		T2	3.26	-	-
14	Curved	T1	-	1.08	102.7
		T2	-	1.02	105.9
15	Straight	T1	3.92	-	-
		T2	3.47	-	-
16	Curved	T1	-	3.83	24.2
		T2	-	3.20	24.1
17	Straight	T1	6.00	-	-
		T2	5.00	-	-

Table A.2: Repeatability Test: T-Spring Geometries

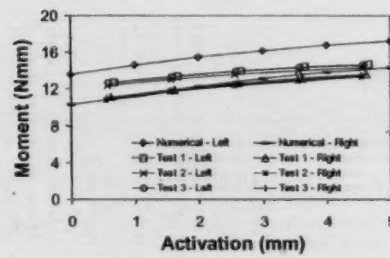
Spring	Section 1 (mm)	2 (mm) (°)	3 (mm)	4 (mm) (°)	5 (mm)	6 (mm) (°)	7 (mm)	8 (mm) (°)	9 (mm)
T3	variable	2.42 32.6	3.89	1.05 104.0	2.79	0.85 74.6	3.20	1.00 139.3	7.88
T4	variable	1.14 25.6	4.01	1.09 111.2	3.19	1.02 74.4	3.31	0.91 139.7	7.94
T5	variable	2.14 25.6	3.95	1.04 108.1	3.13	1.13 83.4	3.15	0.90 146.0	8.04
T6	variable	1.00 29.6	4.64	0.93 105.0	2.81	1.04 76.8	3.36	0.91 140.4	8.09
T7	variable	0.91 29.0	3.73	1.15 100.5	2.72	1.05 76.0	3.00	0.93 138.9	8.19
Spring	Section 10 (°)	11 (mm)	12 (mm) (°)	13 (mm)	14 (mm) (°)	15 (mm)	16 (mm) (°)	17 (mm)	
T3	0.85 145.0	3.28	1.49 75.4	3.14	1.03 110.2	3.84	3.02 28.0	variable	
T4	0.90 141.8	3.49	1.53 79.9	3.09	0.89 103.2	4.04	2.25 29.5	variable	
T5	0.89 142.2	3.31	1.53 76.0	3.14	0.99 106.6	4.37	1.61 28.0	variable	
T6	0.93 143.1	3.51	1.53 79.9	3.40	1.01 104.4	3.69	2.18 26.4	variable	
T7	0.84 144.0	3.51	1.68 79.5	3.21	1.29 108.0	3.35	3.31 23.3	variable	



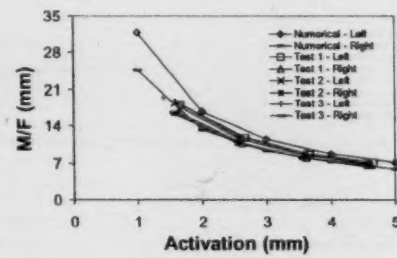
(a) F_z (Tension)



(b) F_x

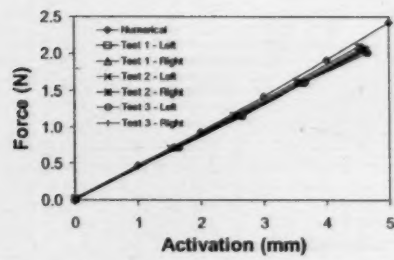


(c) M_y

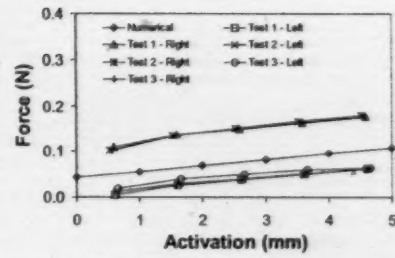


(d) M/F

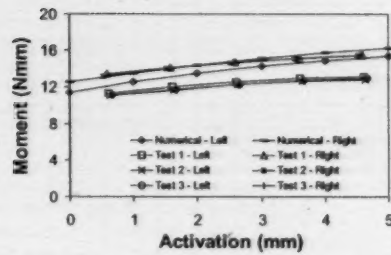
Figure A.1: T1 Planar Results



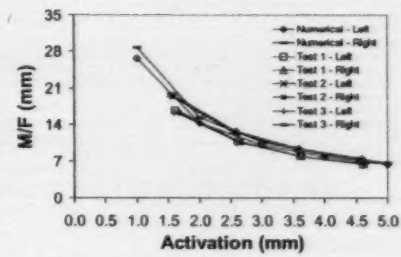
(a) F_z (Tension)



(b) F_x

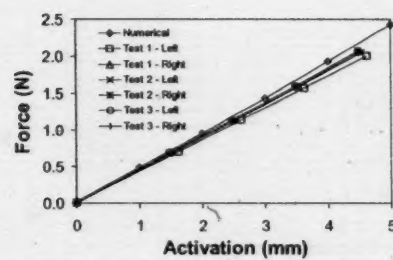


(c) M_y

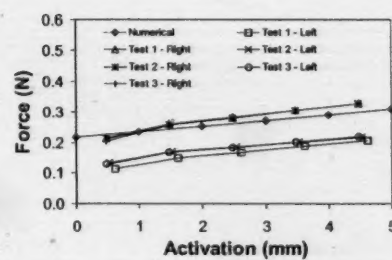


(d) M/F

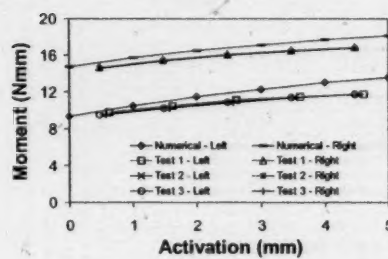
Figure A.2: T1 Results ($x = 1$ mm)



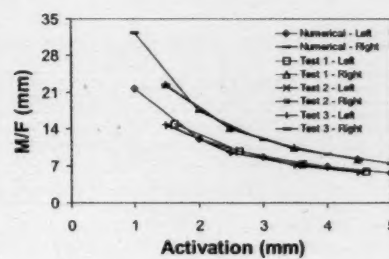
(a) F_z (Tension)



(b) F_x



(c) M_y



(d) M/F

Figure A.3: T1 Results ($x = 2$ mm)

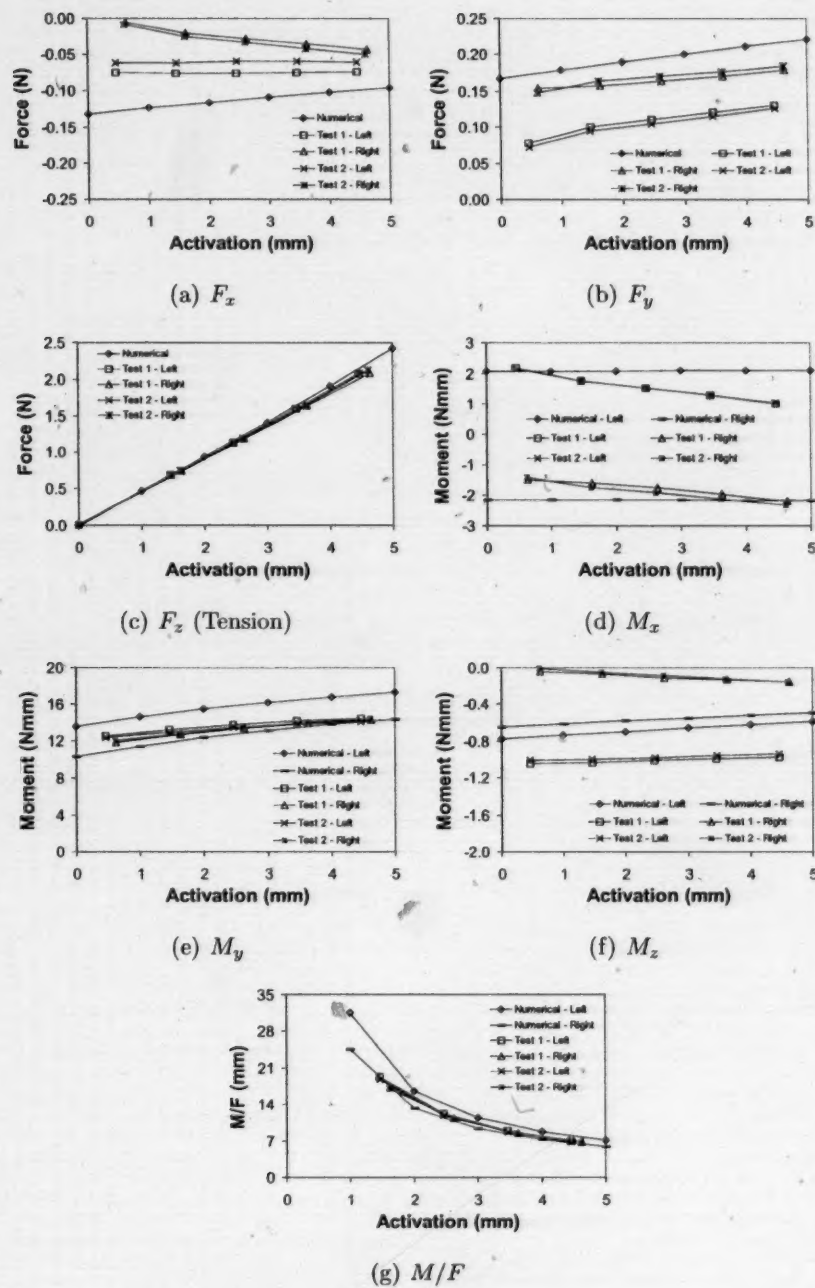
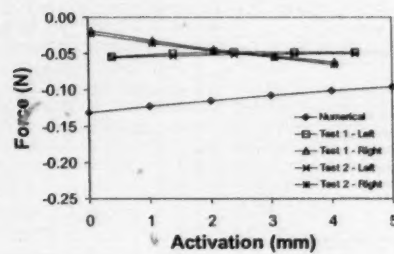
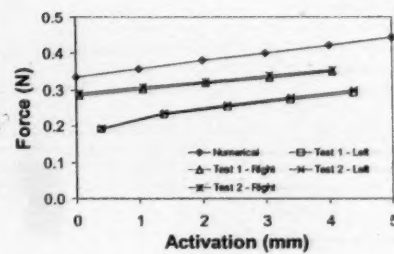


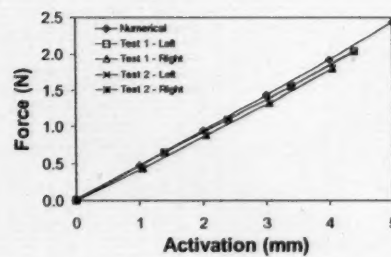
Figure A.4: T1 Results ($y = 1$ mm)



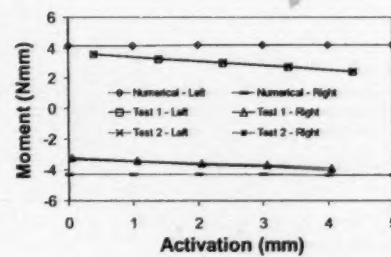
(a) F_x



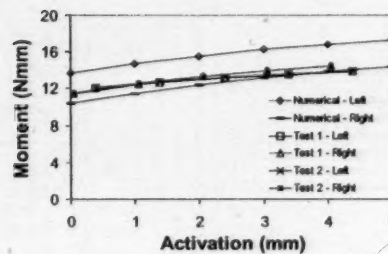
(b) F_y



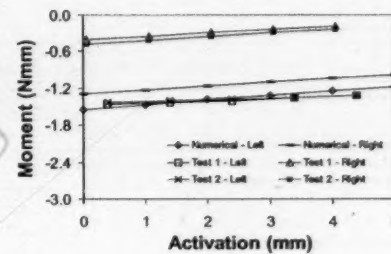
(c) F_z (Tension)



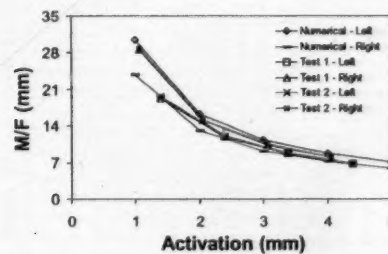
(d) M_x



(e) M_y

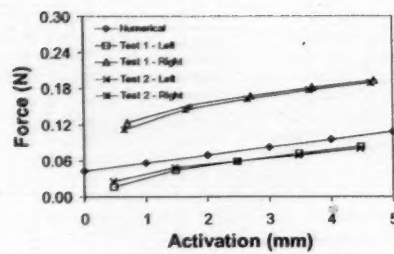


(f) M_z

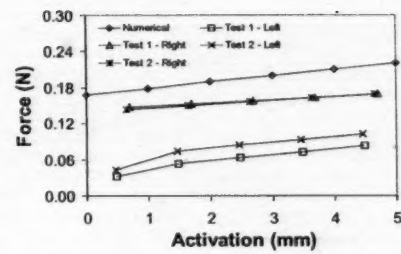


(g) M/F

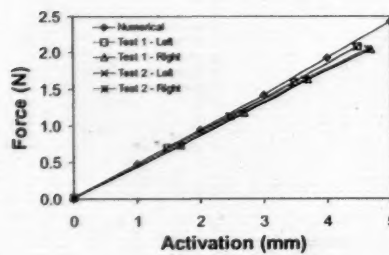
Figure A.5: T1 Results ($y = 2$ mm)



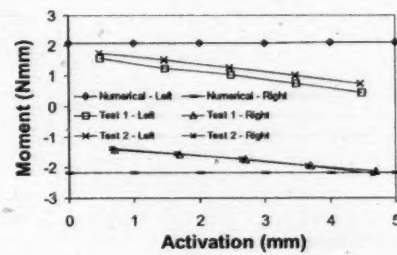
(a) F_x



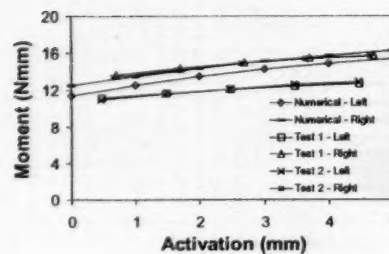
(b) F_y



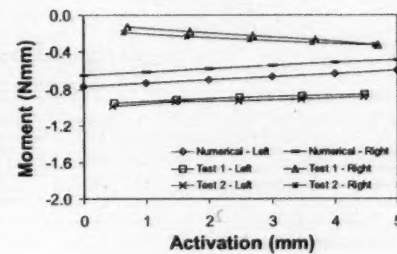
(c) F_z (Tension)



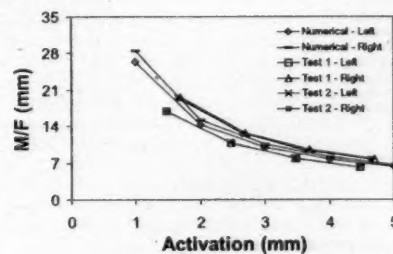
(d) M_x



(e) M_y

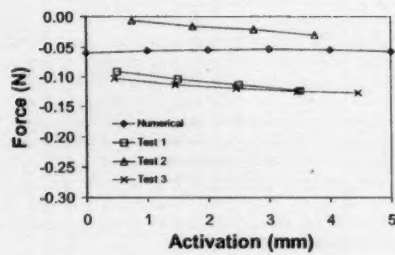


(f) M_z

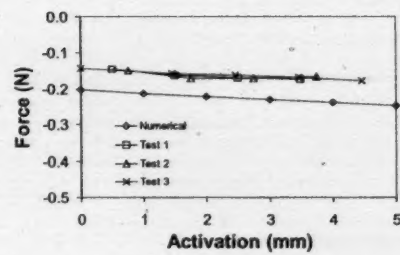


(g) M/F

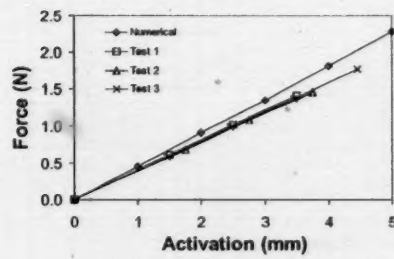
Figure A.6: T1 Results ($x = 1, y = 1$ mm)



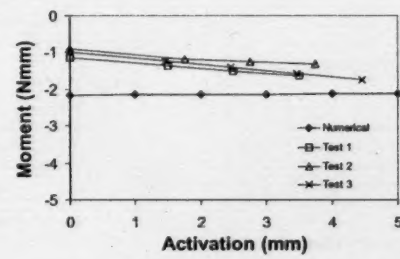
(a) F_x



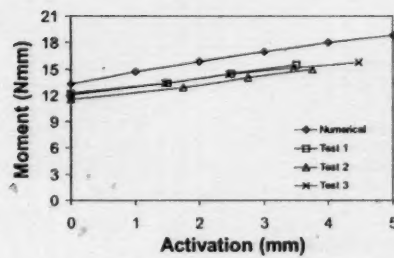
(b) F_y



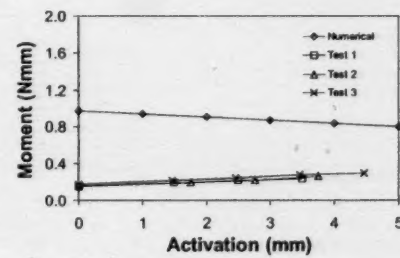
(c) F_z (Tension)



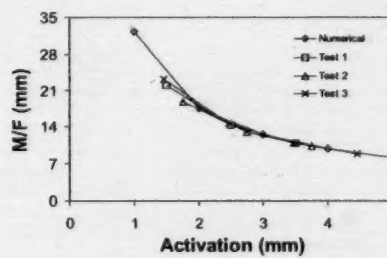
(d) M_x



(e) M_y

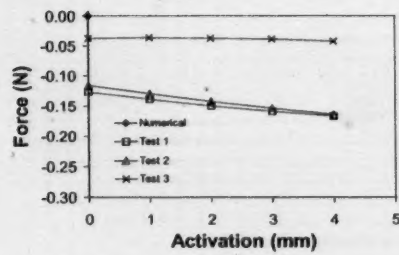


(f) M_z

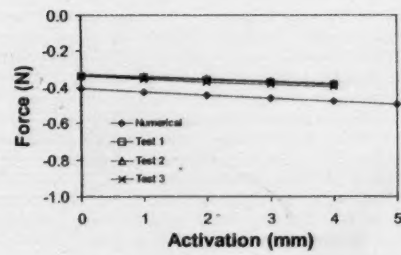


(g) M/F

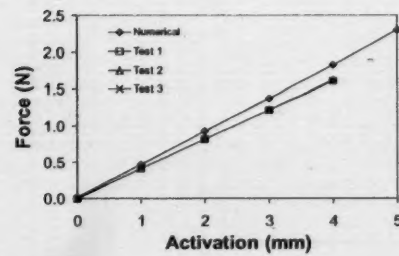
Figure A.7: T2 Results ($y = -1$ mm)



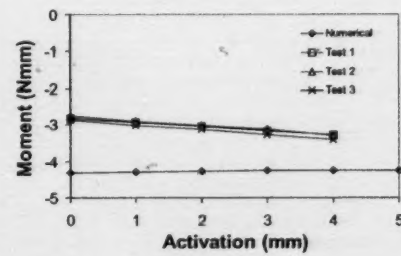
(a) F_x



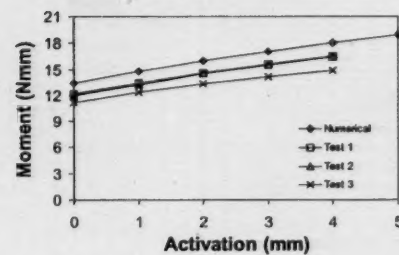
(b) F_y



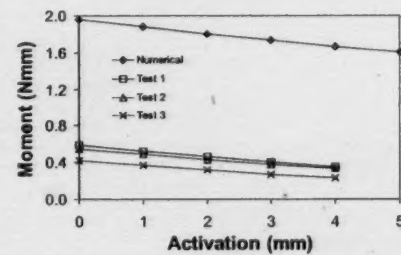
(c) F_z (Tension)



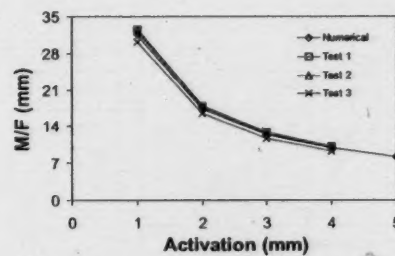
(d) M_x



(e) M_y

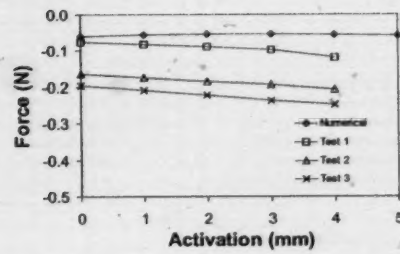


(f) M_z

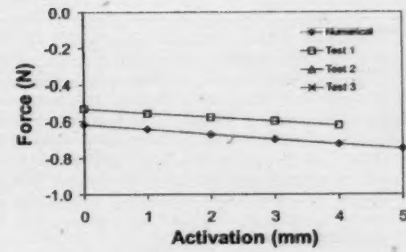


(g) M/F

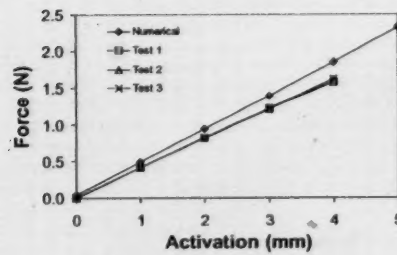
Figure A.8: T2 Results ($y = -2$ mm)



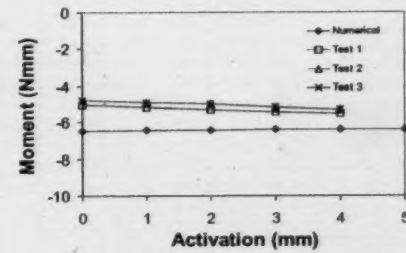
(a) F_x



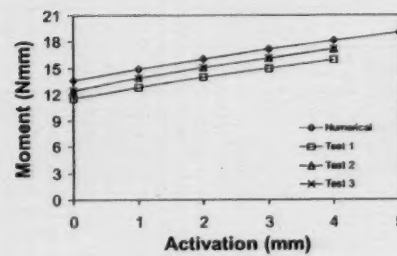
(b) F_y



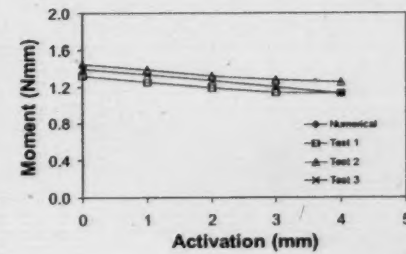
(c) F_z (Tension)



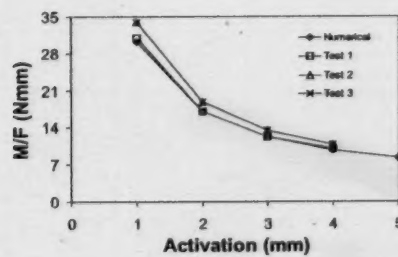
(d) M_x



(e) M_y



(f) M_z



(g) M/F

Figure A.9: T2 Results ($y = -3$ mm)

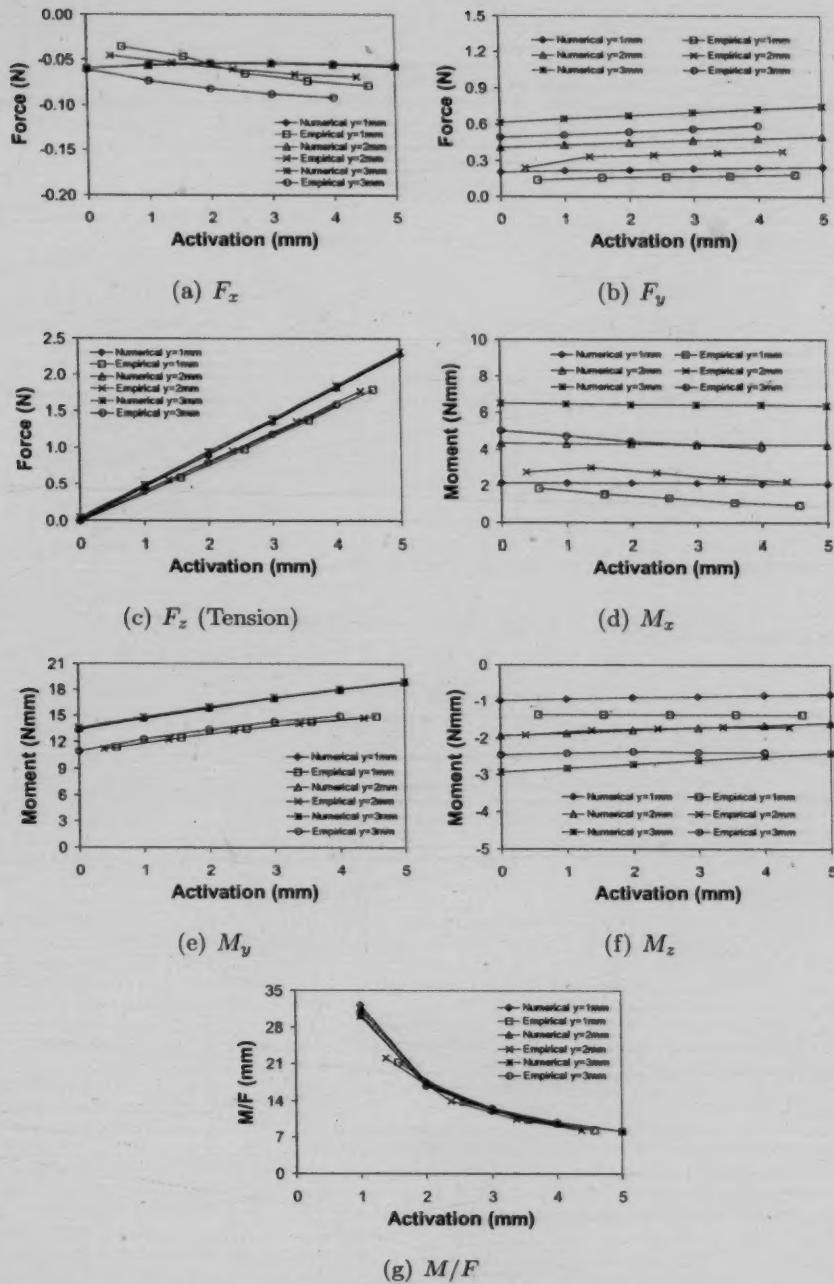
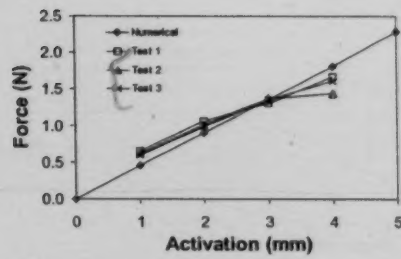
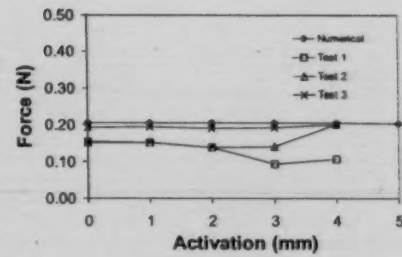


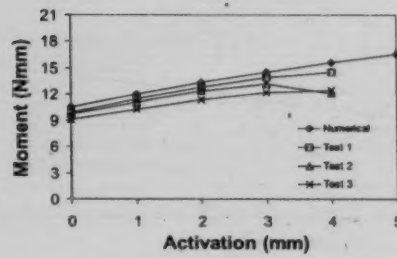
Figure A.10: T2 Results (+y Displacements)



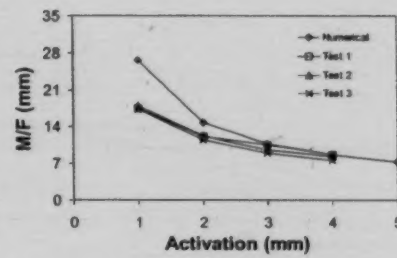
(a) F_z (Tension)



(b) F_x

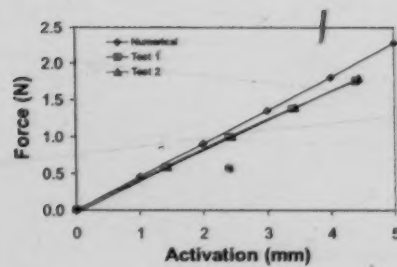


(c) M_y

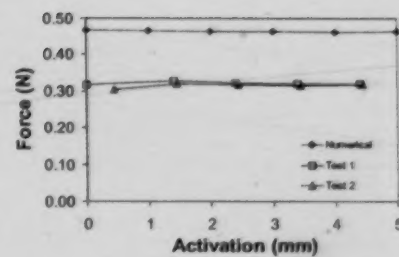


(d) M/F

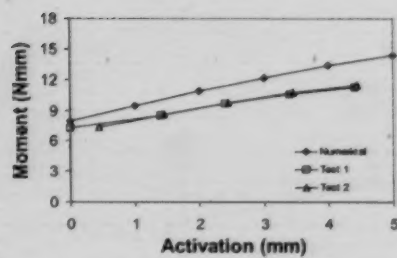
Figure A.11: T2 Results ($x = 1$ mm)



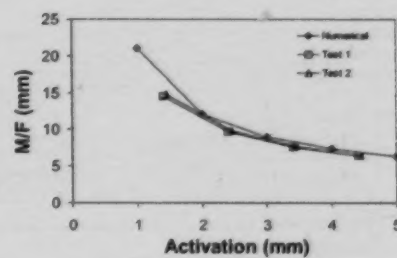
(a) F_z (Tension)



(b) F_x

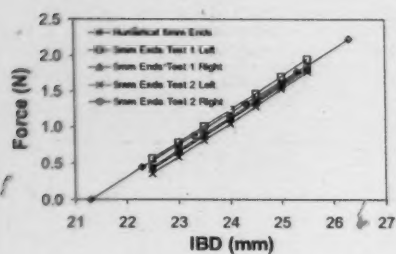


(c) M_y

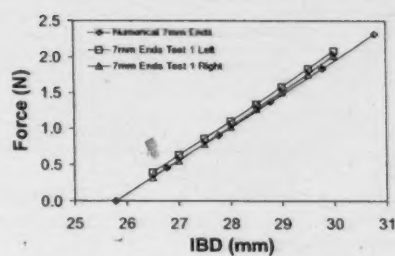


(d) M/F

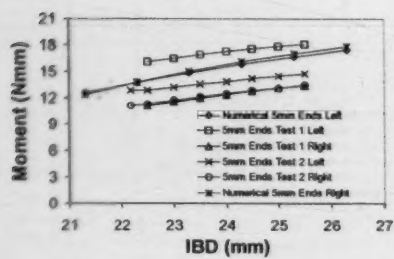
Figure A.12: T2 Results ($x = 2$ mm)



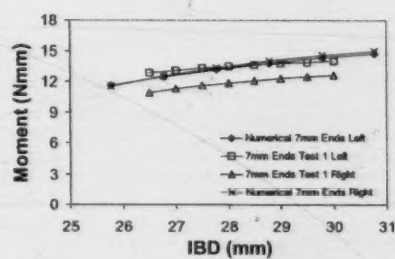
(a) F_z



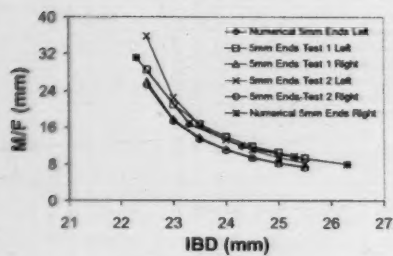
(b) F_z



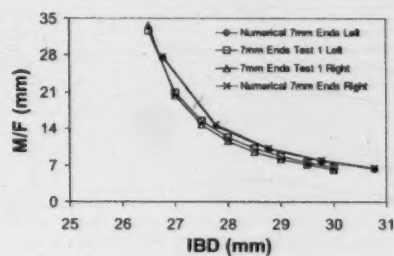
(c) M_y



(d) M_y

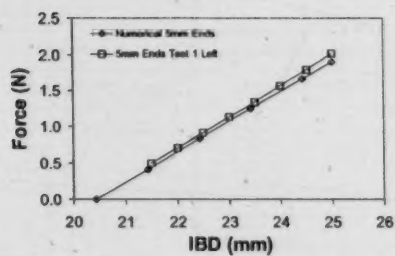


(e) M/F

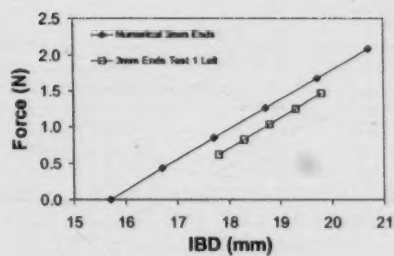


(f) M/F

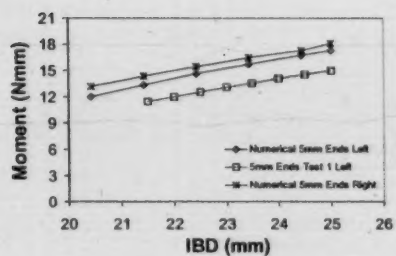
Figure A.13: T3 Planar Results



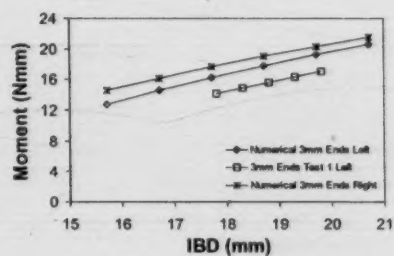
(a) F_z



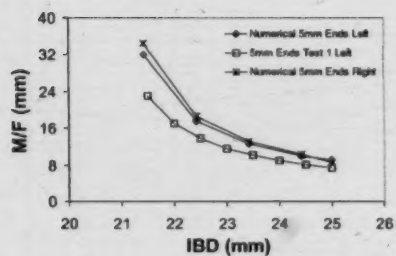
(b) F_z



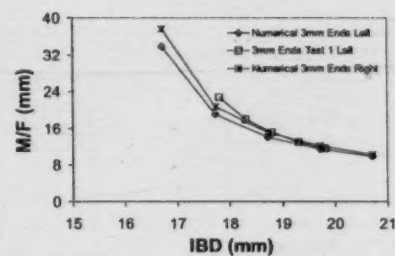
(c) M_y



(d) M_y

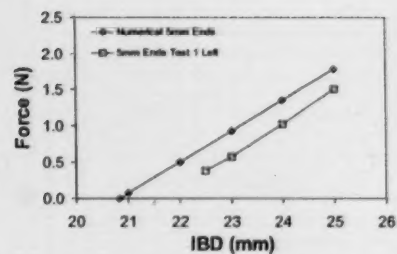


(e) M/F

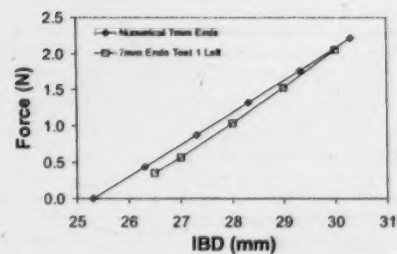


(f) M/F

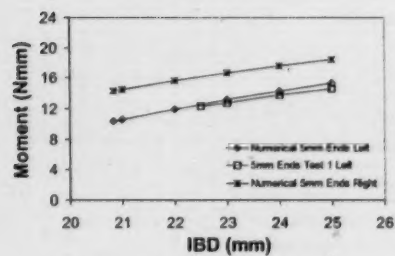
Figure A.14: T4 Planar Results



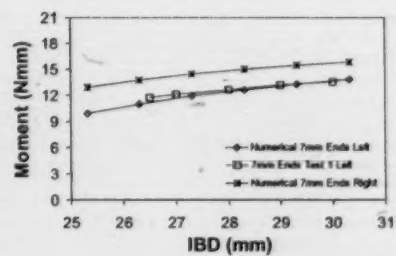
(a) F_z



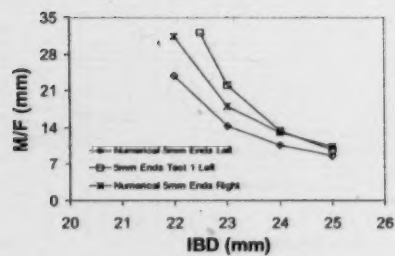
(b) F_z



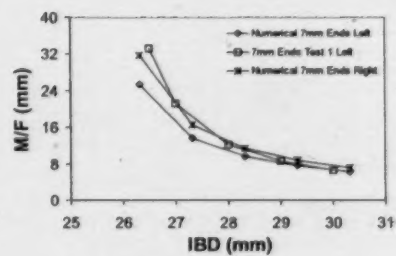
(c) M_y



(d) M_y



(e) M/F



(f) M/F

Figure A.15: T5 Planar Results

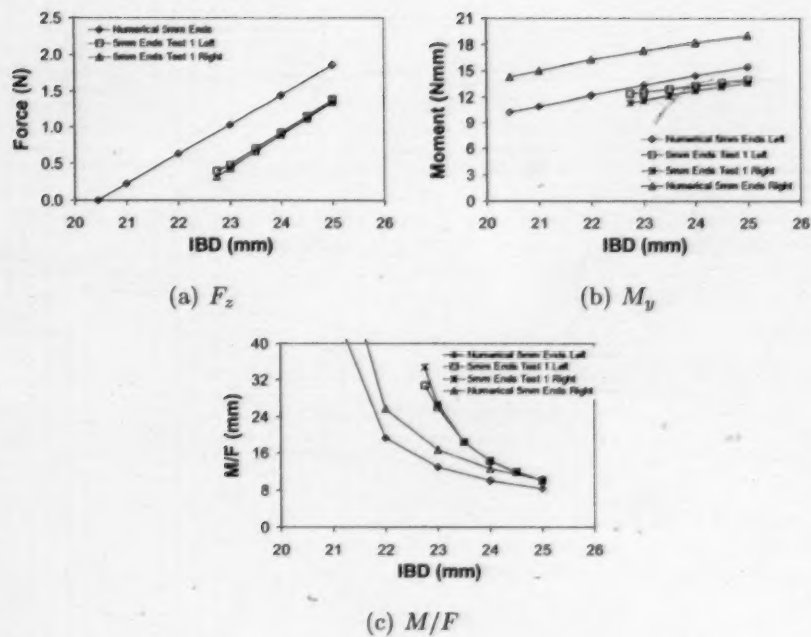
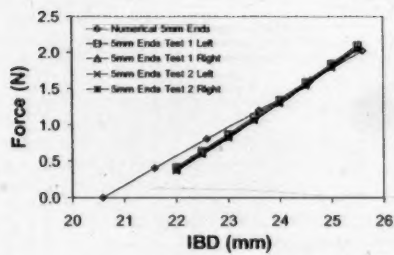
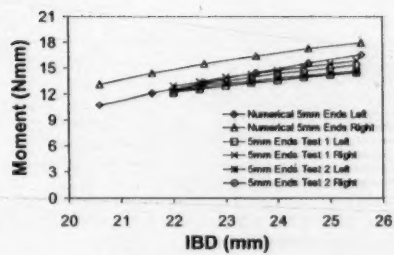


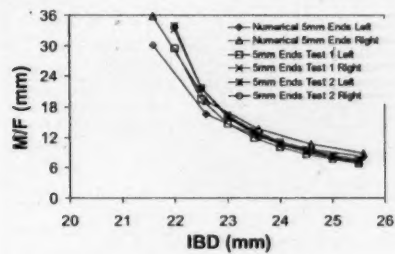
Figure A.16: T6 Planar Results



(a) F_z



(b) M_y



(c) M/F

Figure A.17: T7 Planar Results

A.2 Anti-Rotation T-Springs

Results from the anti-rotation T-spring tests are shown in this section. When necessary, the empirical results are only plotted with the activation shift already included. Tables A.3 shows the anti-rotation T-spring geometries.

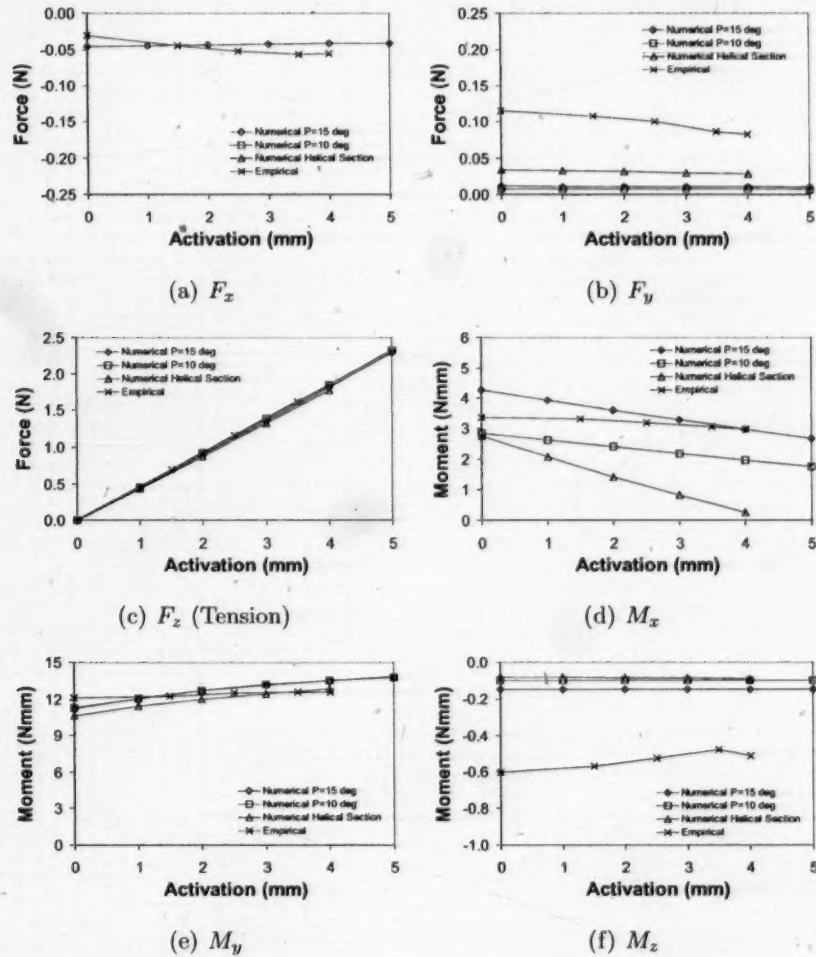


Figure A.18: Anti-Rotation T1 Planar Results

Table A.3: Anti-Rotation T-Spring Geometries

Section	Straight, Curved, or Twisted	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Straight	Anti1	8.00	-	-
		Anti2	4.00	-	-
2	Curved	Anti1	-	2.12	30.2
		Anti2	-	1.82	29.5
3	Straight	Anti1	4.12	-	-
		Anti2	4.14	-	-
4	Curved	Anti1	-	1.04	104.9
		Anti2	-	0.89	109.8
5	Twisted	Anti1	2.84	-	-
		Anti2	3.15	-	12.0
6	Curved	Anti1	-	1.11	80.5
		Anti2	-	1.24	73.3
7	Straight	Anti1	2.70	-	-
		Anti2	3.40	-	-
8	Curved	Anti1	-	1.00	143.9
		Anti2	-	1.00	142.4
9	Straight	Anti1	8.23	-	-
		Anti2	7.73	-	-
10	Curved	Anti1	-	0.80	139.8
		Anti2	-	0.97	140.7
11	Straight	Anti1	3.69	-	-
		Anti2	3.66	-	-
12	Curved	Anti1	-	1.20	79.4
		Anti2	-	1.63	73.0
13	Twisted	Anti1	3.26	-	-
		Anti2	3.16	-	12.0
14	Curved	Anti1	-	1.02	105.9
		Anti2	-	1.00	109.6
15	Straight	Anti1	3.47	-	-
		Anti2	3.94	-	-
16	Curved	Anti1	-	3.20	24.1
		Anti2	-	1.30	28.4
17	Straight	Anti1	8.00	-	-
		Anti2	6.00	-	-

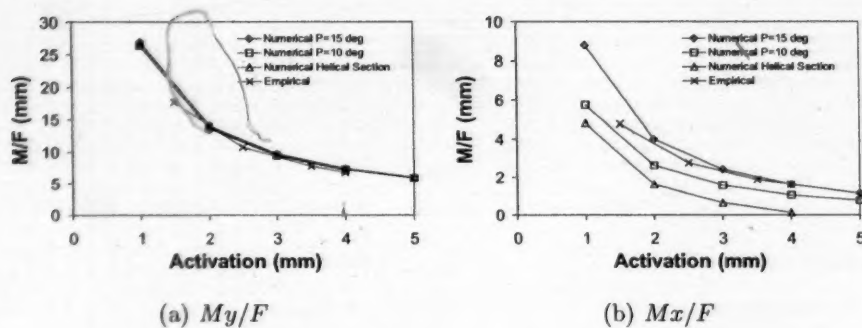


Figure A.19: Anti-Rotation T1 Moment-to-Forces Ratios

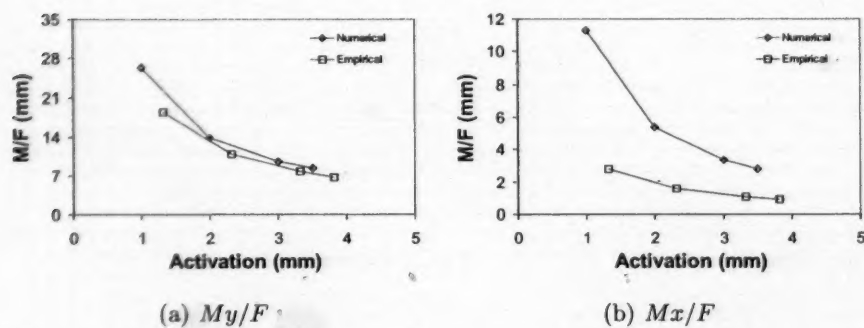
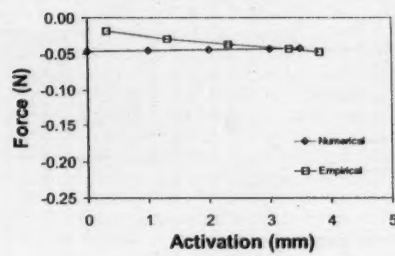
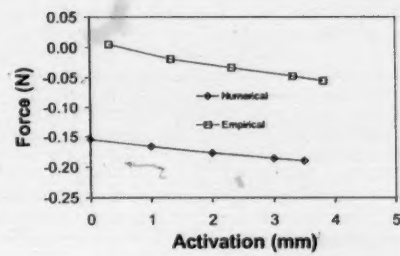


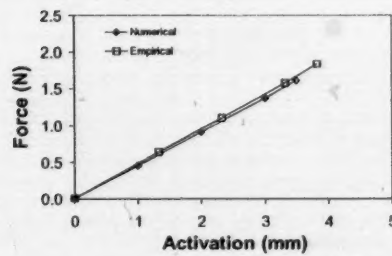
Figure A.20: Anti-Rotation T1 Moment-to-Forces Ratios ($y = 1$ mm)



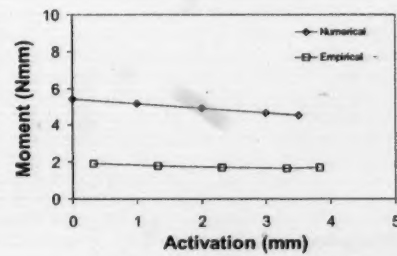
(a) F_x



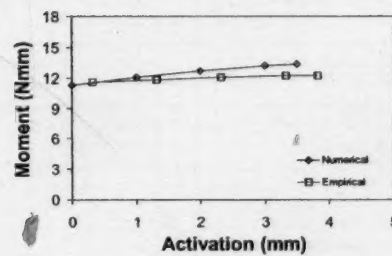
(b) F_y



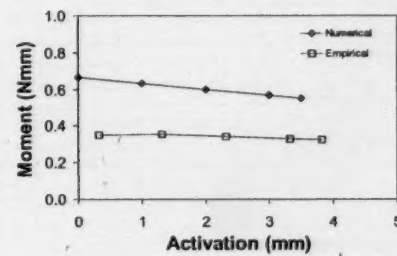
(c) F_z (Tension)



(d) M_x

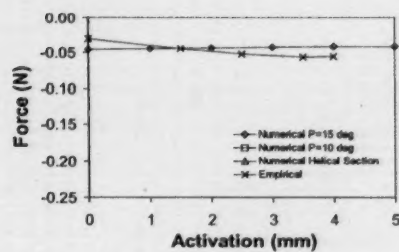


(e) M_y

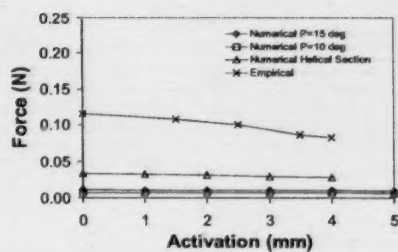


(f) M_z

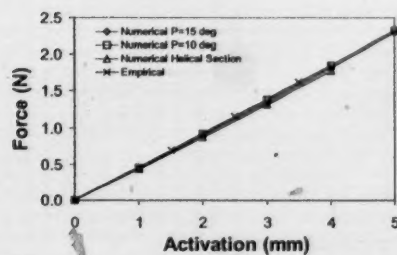
Figure A.21: Anti-Rotation T1 Results ($y = 1$ mm)



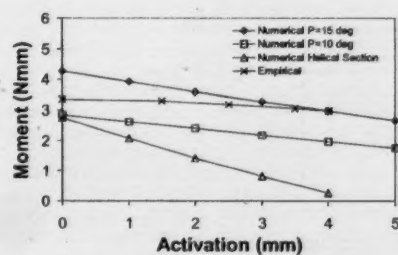
(a) F_x



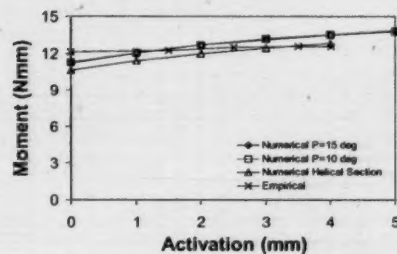
(b) F_y



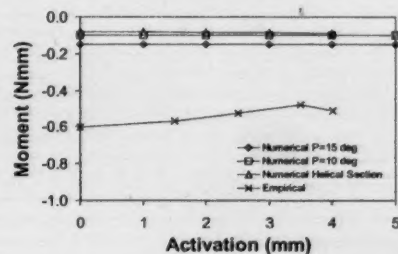
(c) F_z (Tension)



(d) M_x



(e) M_y



(f) M_z

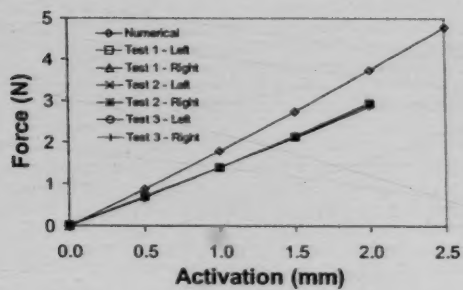
Figure A.22: Anti-Rotation T2 Planar Results

A.3 Vertical-Loop Springs

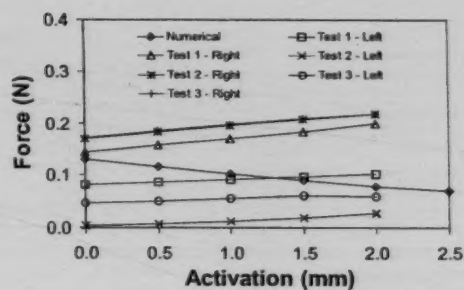
Results from the vertical-loop spring tests are shown in this section. Tables A.4 shows the vertical-loop spring geometries. It should also be noted that the left and right ends were not tested for every spring.

Table A.4: V-Spring Geometries

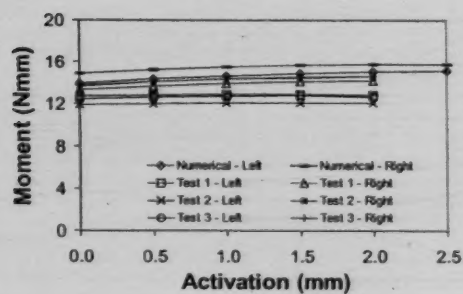
Section	Straight or Curved	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Straight	V1	7.00	-	-
		V2	7.00	-	-
2	Curved	V1	-	1.16	514.1
		V2	-	1.08	514.8
3	Straight	V1	4.08	-	-
		V2	4.23	-	-
4	Curved	V1	-	1.07	164.5
		V2	-	1.04	165.8
5	Straight	V1	4.21	-	-
		V2	3.96	-	-
6	Curved	V1	-	1.15	515.8
		V2	-	1.14	514.3
7	Straight	V1	7.00	-	-
		V2	7.00	-	-



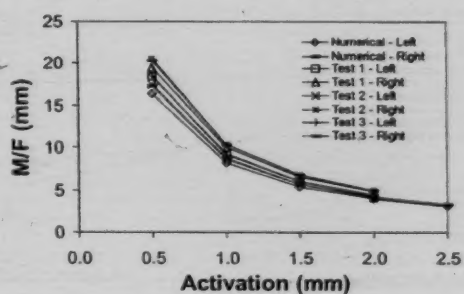
(a) F_z (Tension)



(b) F_x

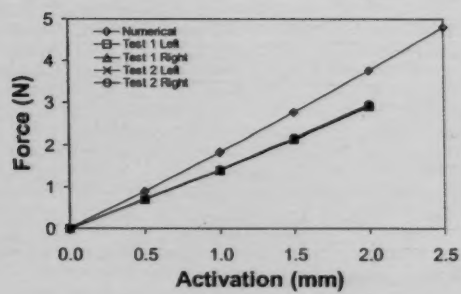


(c) M_y

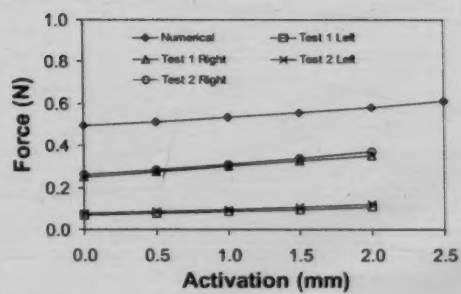


(d) M/F

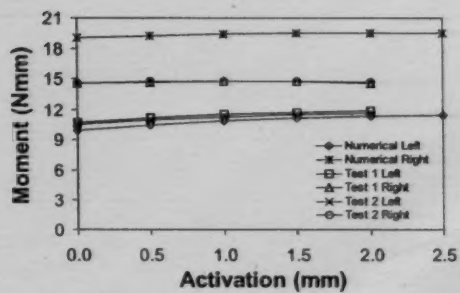
Figure A.23: V1 Planar Results



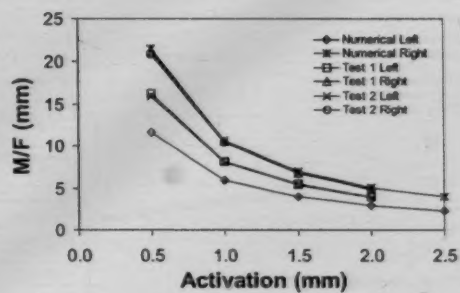
(a) F_z (Tension)



(b) F_x

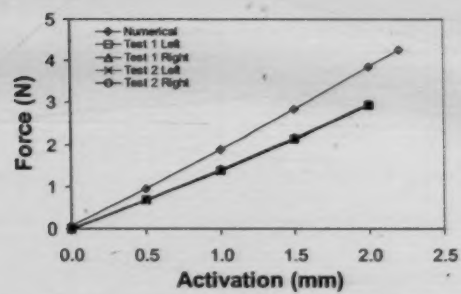


(c) M_y

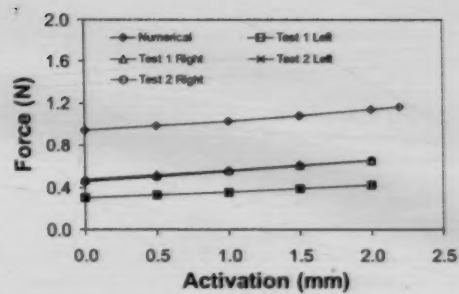


(d) M/F

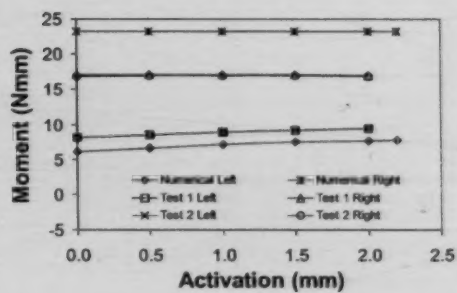
Figure A.24: V1 Results ($x = 1$ mm)



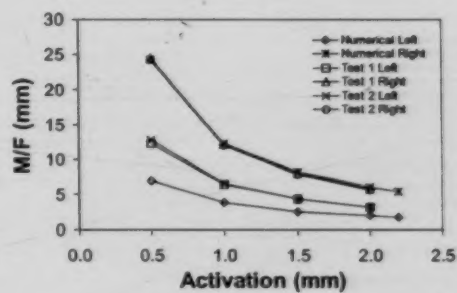
(a) F_z (Tension)



(b) F_x

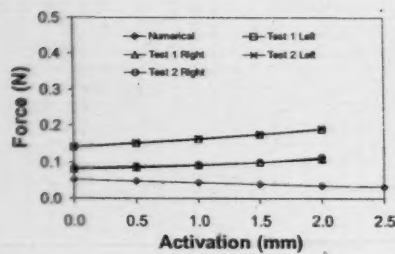


(c) M_y

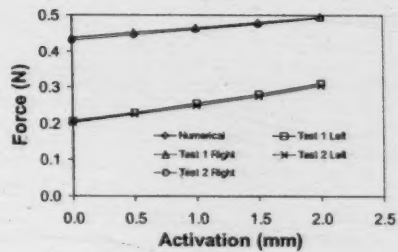


(d) M/F

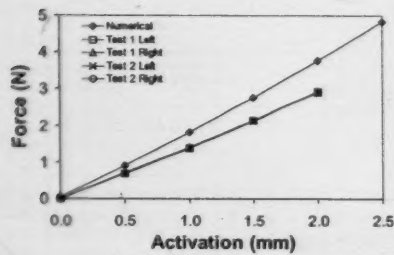
Figure A.25: V1 Results ($x = 2$ mm)



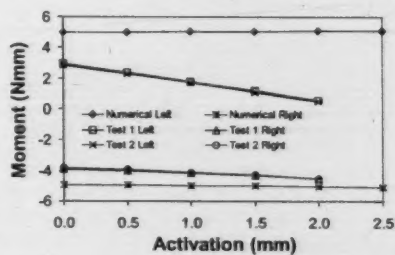
(a) F_x



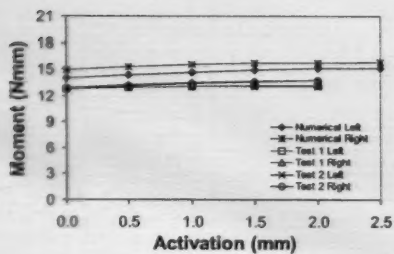
(b) F_y



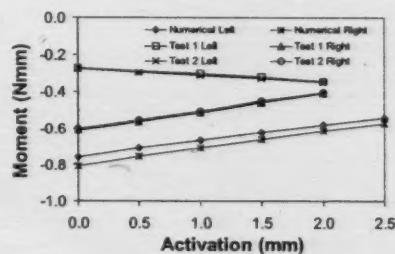
(c) F_z (Tension)



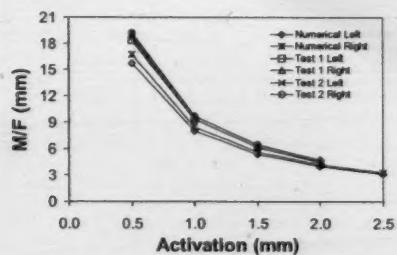
(d) M_x



(e) M_y



(f) M_z



(g) M/F

Figure A.26: V1 Results ($y = 1$ mm)

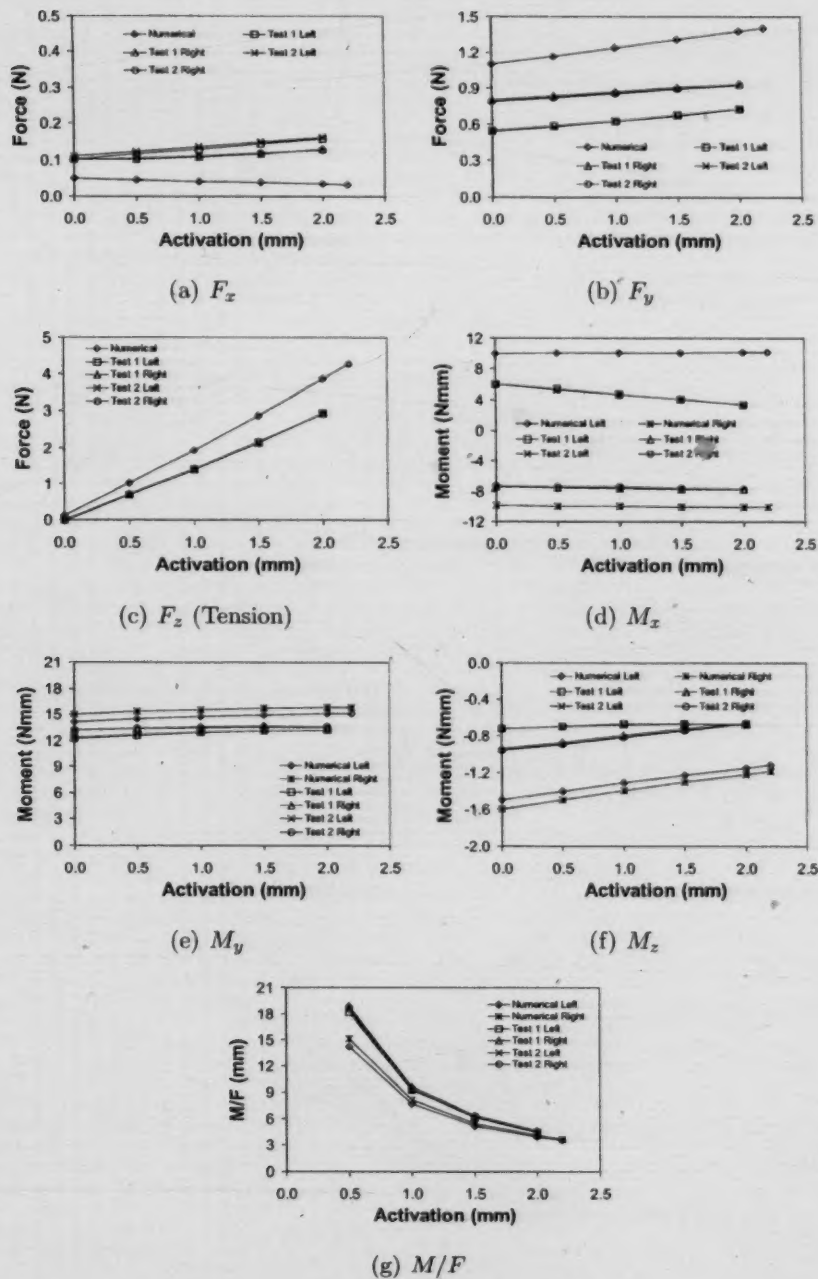


Figure A.27: V1 Results ($y = 2$ mm)

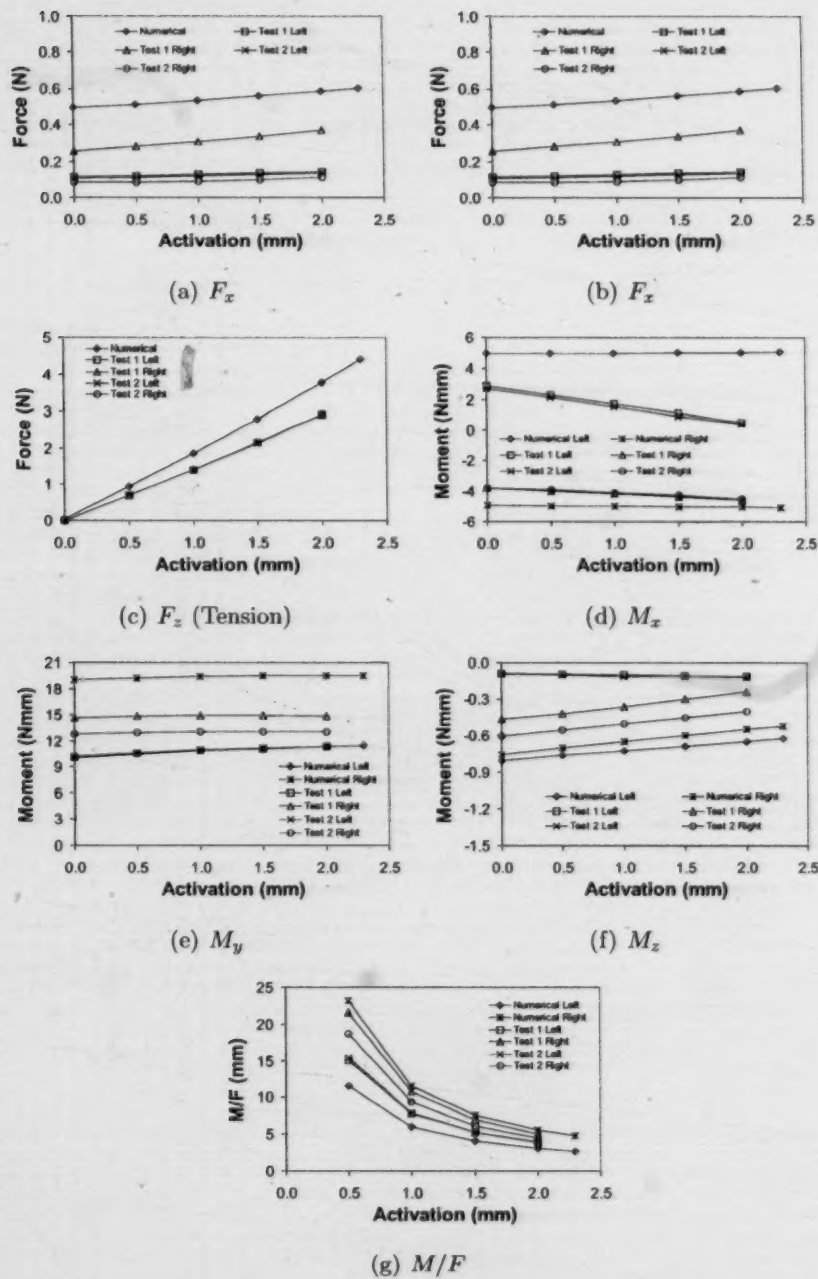


Figure A.28: V1 Results ($x = 1, y = 1$ mm)

A.4 Archial-Loop Springs

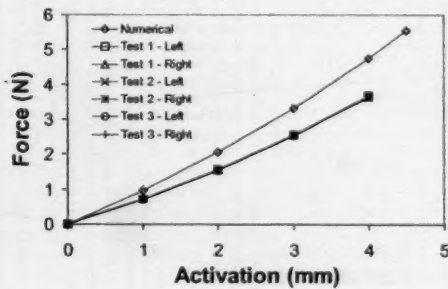
Results from the archial-loop spring tests are shown in this section. Tables A.5 and A.6 show the archial-loop spring geometries. It should also be noted that the left and right ends were not tested for every spring.

Table A.5: A-Spring Geometries

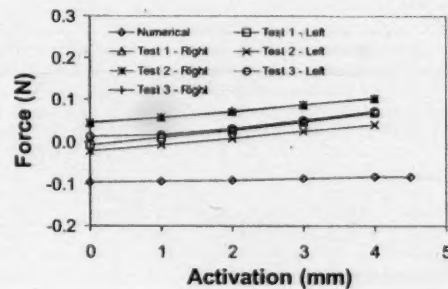
Section	Straight or Curved	Spring	Length (mm)	Radius (mm)	Angle (°)
1	Curved	A1	-	14.30	variable
		A2	-	14.72	variable
2	Curved	A1	-	1.06	481.3
		A2	-	1.01	473.3
3	Straight	A1	5.59	-	-
		A2	5.60	-	-
4	Curved	A1	-	1.11	165.0
		A2	-	1.12	161.0
5	Straight	A1	5.01	-	-
		A2	5.10	-	-
6	Curved	A1	-	1.04	482.6
		A2	-	1.04	476.9
7	Curved	A1	-	14.81	variable
		A2	-	15.16	variable

Table A.6: Repeatability Test: A-Spring Geometries

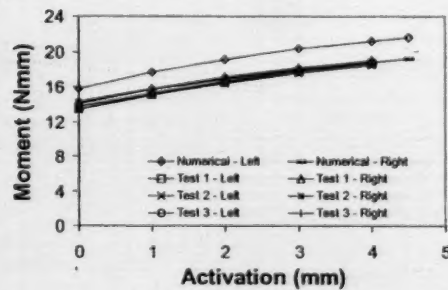
Spring	Section 1 (mm) (°)	2 (mm) (°)	3 (mm)	4 (mm) (°)	5 (mm)	6 (mm) (°)	7 (mm) (°)
A3	16.48 variable	1.08 487.6	5.43	1.03 154.4	5.70	0.96 488.1	19.71 variable
A4	16.11 variable	1.03 489.5	5.88	0.94 157.4	5.71	1.08 486.5	16.79 variable
A5	14.49 variable	1.01 481.6	5.33	0.88 153.4	5.92	1.01 475.6	15.09 variable
A6	15.11 variable	1.03 480.3	5.79	0.88 154.2	5.62	1.10 489.6	16.04 variable
A7	16.11 variable	1.04 483.1	5.80	0.92 154.5	5.24	1.09 482.0	16.88 variable



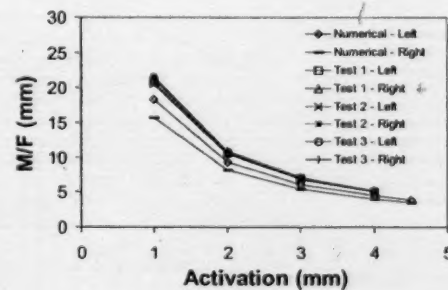
(a) F_z (Tension)



(b) F_x

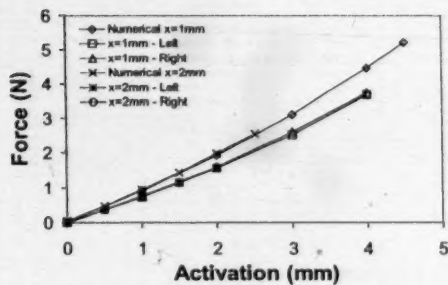


(c) M_y

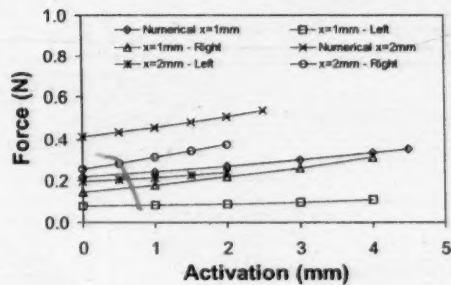


(d) M/F

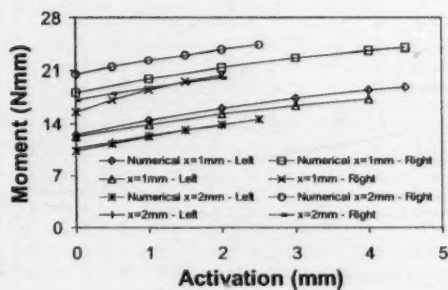
Figure A.29: A1 Planar Results



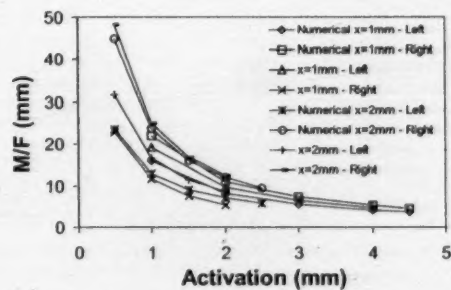
(a) F_z (Tension)



(b) F_x

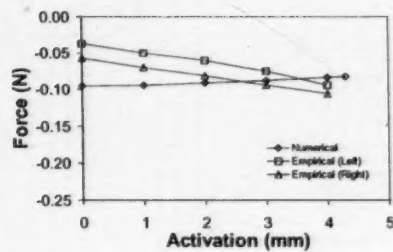


(c) M_y

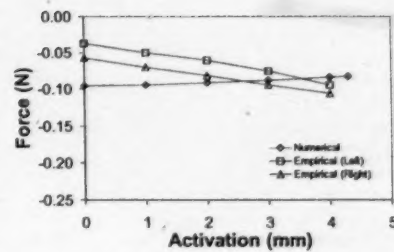


(d) M/F

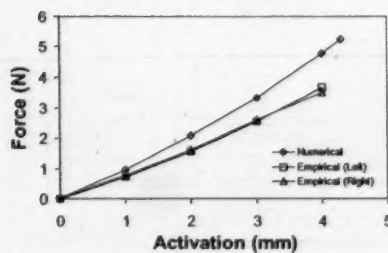
Figure A.30: A1 Results (+x Displacements)



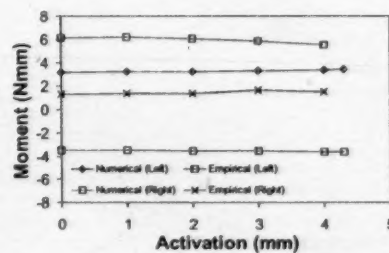
(a) F_x



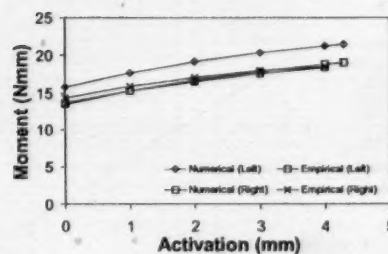
(b) F_x



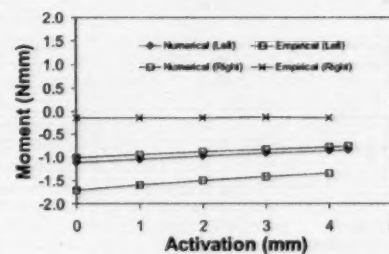
(c) F_z (Tension)



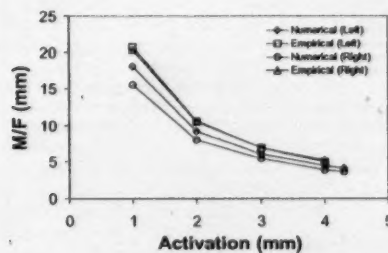
(d) M_x



(e) M_y

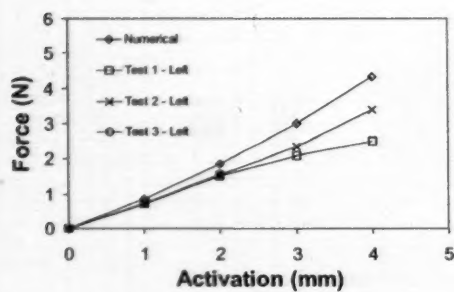


(f) M_z

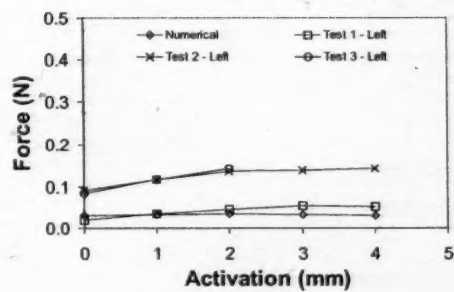


(g) M/F

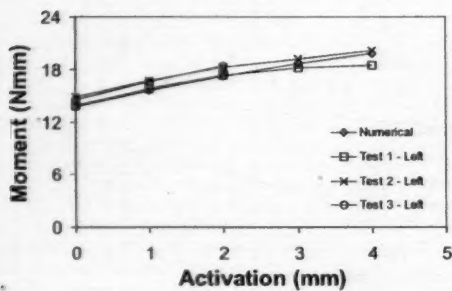
Figure A.31: A1 Results ($y = 1$ mm)



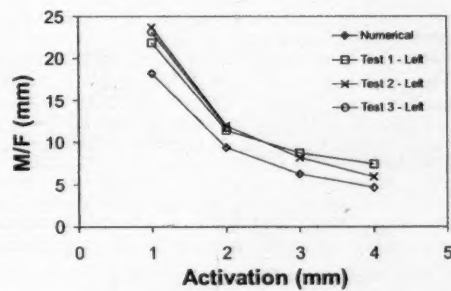
(a) F_z (Tension)



(b) F_x

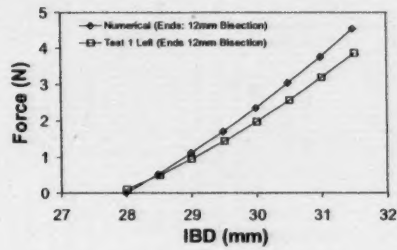


(c) M_y

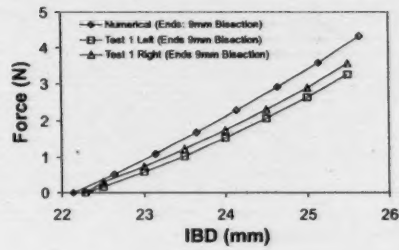


(d) M/F

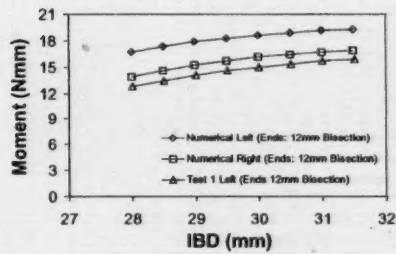
Figure A.32: A2 Planar Results



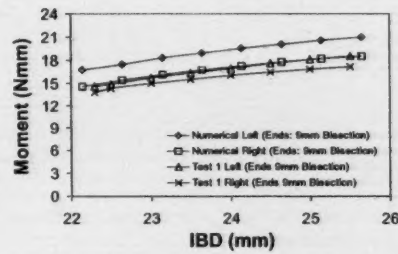
(a) F_z



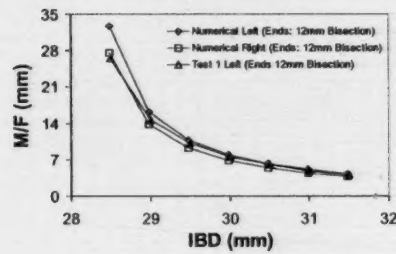
(b) F_z



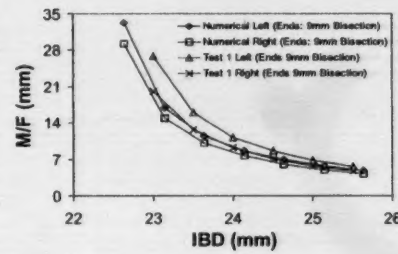
(c) M_y



(d) M_y

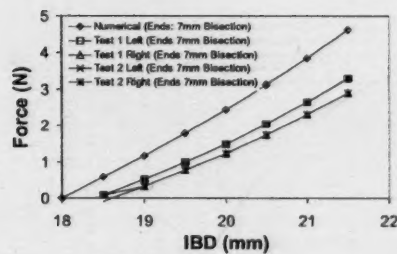


(e) M/F

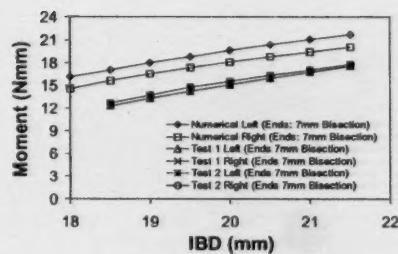


(f) M/F

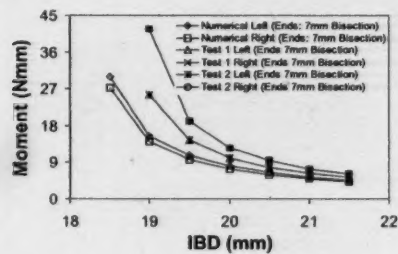
Figure A.33: A3 Planar Results



(a) F_z

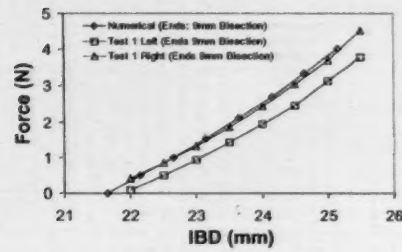


(b) M_y

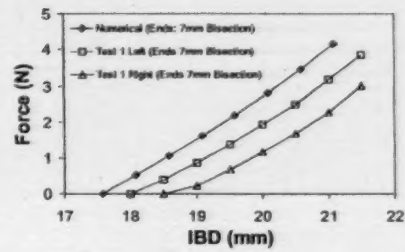


(c) M/F

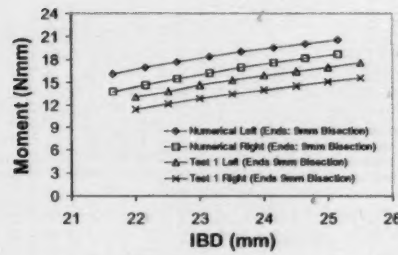
Figure A.34: A3 Planar Results Continued



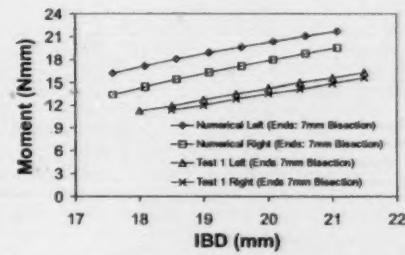
(a) F_z



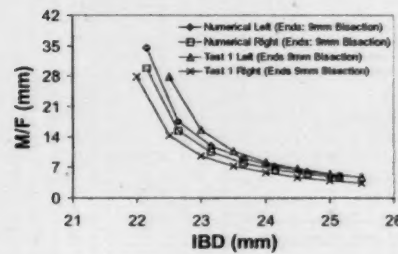
(b) F_z



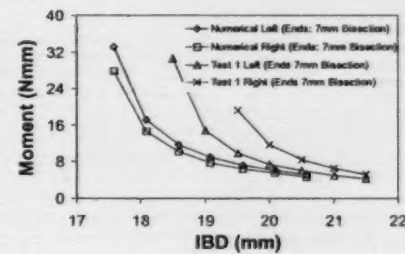
(c) M_y



(d) M_y

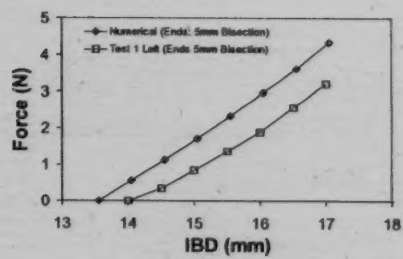


(e) M/F

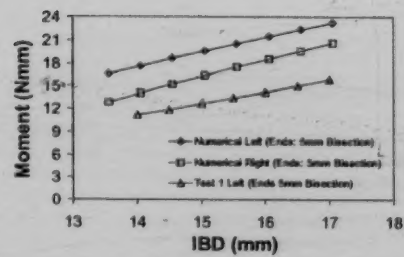


(f) M/F

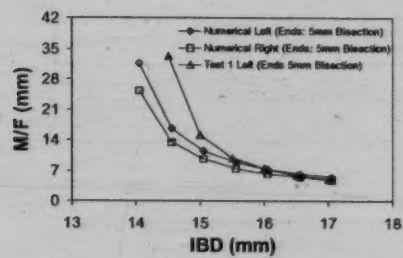
Figure A.35: A4 Planar Results



(a) F_z

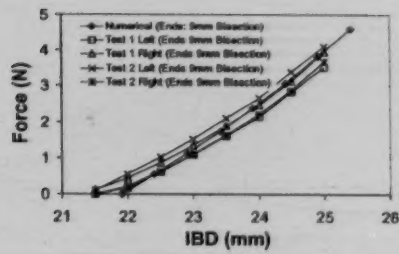


(b) M_y

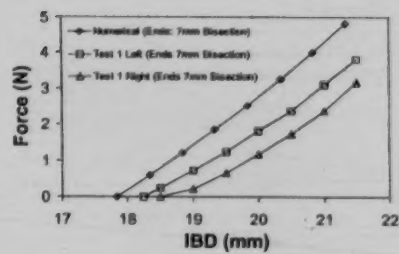


(c) M/F

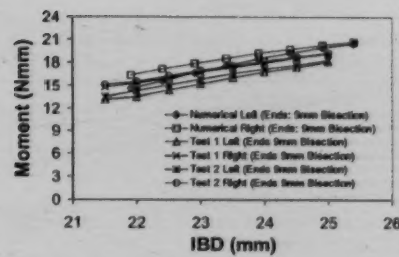
Figure A.36: A4 Planar Results Continued



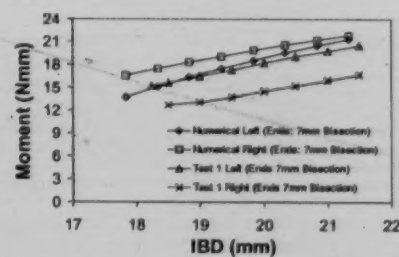
(a) F_z



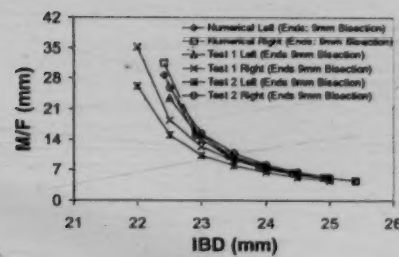
(b) F_z



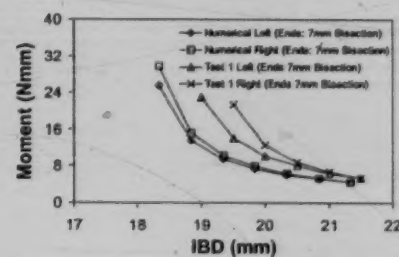
(c) M_y



(d) M_y

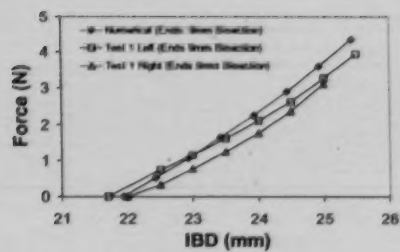


(e) M/F

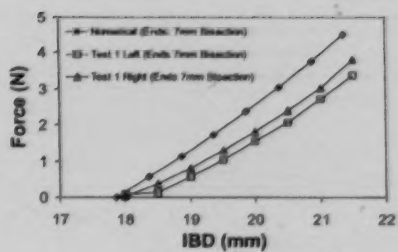


(f) M/F

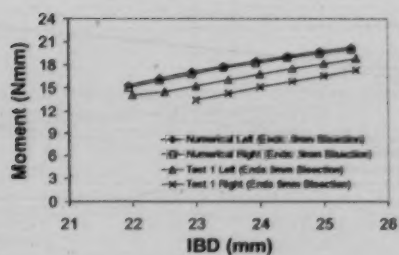
Figure A.37: A5 Planar Results



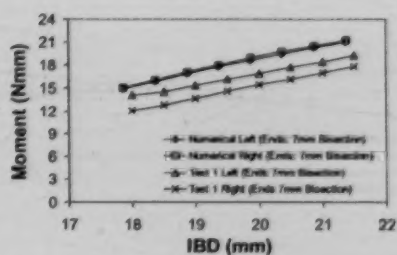
(a) F_z



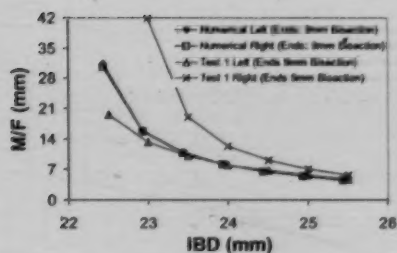
(b) F_z



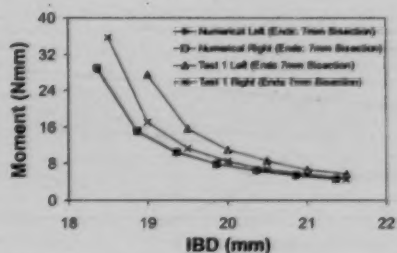
(c) M_y



(d) M_y

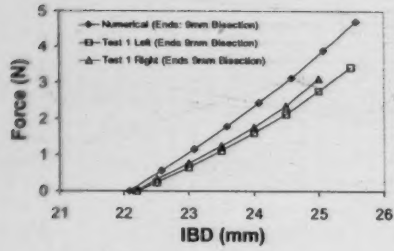


(e) M/F

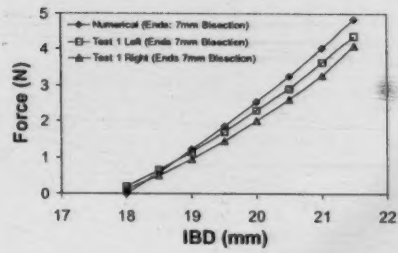


(f) M/F

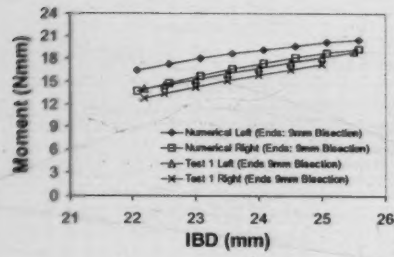
Figure A.38: A6 Planar Results



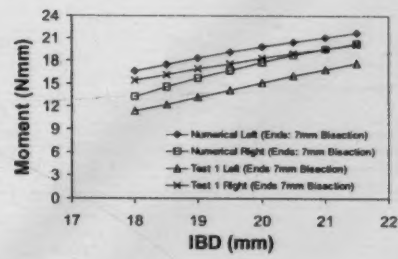
(a) F_z



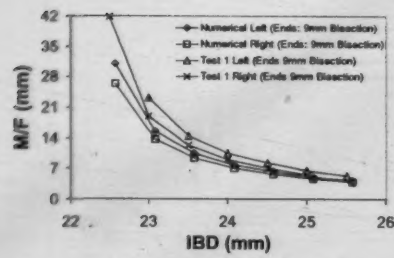
(b) F_z



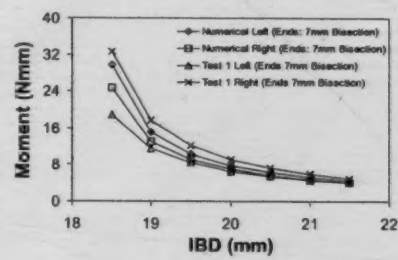
(c) M_y



(d) M_y



(e) M/F



(f) M/F

Figure A.39: A7 Planar Results

Appendix B

Radius Calculation Derivation

This appendix contains the derivation of the formula used to calculate the radius of the curved sections of the digitized geometry. Figure B.1 illustrates a typical curved section of an orthodontic spring. To define the curved section numerically, both R and θ are required. When measuring the empirical springs however, R was unable to be directly measured. Only values for θ , c and h could be directly measured. θ was obtained by measuring the change in angle between the straight sections at the ends of the curved section. c and h were measured directly. Below is a step by step outline of how to determine R from the known variables.

Using R (the radius of the curve) as the hypotenuse of a right-angled triangle it can be written that:

$$R^2 = d^2 + \left(\frac{1}{2}c\right)^2$$

Solving for c^2 gives:

$$c^2 = 4(R^2 - d^2)$$

Expanding $(R^2 - d^2)$ gives:

$$c^2 = 4(R - d)(R + d)$$

But we know that:

$$h = R - d$$

And that $R + d$ can be written as:

$$R + d = 2R - h$$

Therefore,

$$c^2 = 4h(2R - h)$$

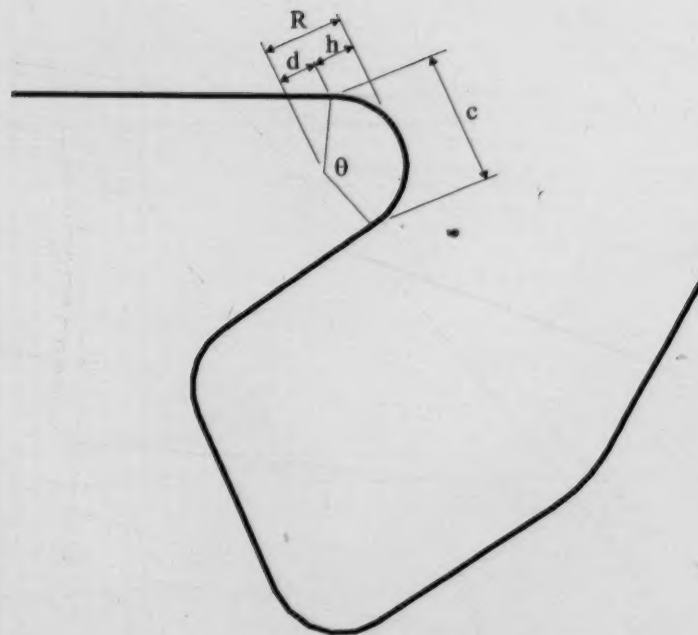


Figure B.1: Curved Section Radius Calculation Derivation

Rearranging the above equation gives:

$$8Rh = c^2 + 4h^2$$

Solving for R gives:

$$R = \frac{1}{8h}(c^2 + 4h^2)$$

Error in R :

$$\begin{aligned} \varepsilon R &= \frac{\partial R}{\partial h} \varepsilon h + \frac{\partial R}{\partial c} \varepsilon c \\ \varepsilon R &= \left(\frac{-c^2}{8h^2} + \frac{1}{2} \right) \varepsilon h + \frac{c}{4h} \varepsilon c \end{aligned}$$